A perfectness concept for multicriteria games

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Abstract. This paper considers a refinement of equilibria for multicriteria games based on the perfectness concept of Selten (1975). Existence of perfect equilibrium points is shown and several characterizations are provided. Furthermore, contrary to the result for equilibria for multicriteria games, an example shows that there is no exact correspondence between perfect equilibrium points and the perfect Nash equilibria of the related trade-off games.

Key words: Multicriteria games, perfect equilibria, trade-off games

1 Introduction

Interactive decision situations with more than one decision maker and in which multiple objectives play a role, can be modelled by means of multicriteria games, an alternative name for games with vector payoffs.

A pioneering paper which deals with multicriteria games is Blackwell (1956). Here an analog of the minimax theorem is provided for repeated zero-sum games with vector payoffs based on the concepts of approachability/excludability of subsets of payoff vectors. Shapley (1959) defined the notion of equilibrium points for (one-shot) two-person games with vector payoffs and showed the correspondence between equilibria and Nash equilibria of so-called trade-off games.


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Up to now multicriteria analysis has attracted relatively little attention in the literature on games although the decision theoretic counterpart w.r.t. multiobjective programming is rather well-developed. We refer to Cochrane and Zeleny (1973), Cohon (1978) and more recently French et al. (1983), Chankong and Haimes (1983), Steuer (1986) and Vincke (1992). In our opinion multicriteria games can be of use in modelling various real-life situations where several objectives have to be taken into account such as in politics and management decisions, especially in situations in which the agents do not have an a priori opinion on the relative importance of the components of their payoff vectors.

This paper aims for a refinement theory for (weak) equilibria for non zero-sum multi-criteria games based on ideas and insights of the extensive literature on refinements for Nash equilibria in unicriterium games (cf. van Damme (1991)). In particular we introduce perfect equilibrium points in the sense of Selten (1975). Throughout the paper an example of a multicriteria production-inspection game illustrates the notions of equilibrium points, perfect equilibrium points and related trade-off games.

Section 2 contains the necessary definitions on multicriteria games and equilibrium points and recalls the correspondence between equilibria and Nash equilibria of related trade-off games.

Section 3 extends the characterization of a Nash equilibrium, the carriers being subsets of the best reply sets, to equilibrium points for multicriteria games in the sense that the carriers have to be subsets of a so-called efficient pure best reply sets. This characterization also reveals the possibility to order the strategies by means of levels of best reply sets, thus providing a first indication towards a properness concept a la Myerson (1978). The definition of perfect equilibria by means of perturbed games is generalized towards multicriteria games in section 4. Existence of perfect equilibrium points is shown using the characterization of equilibrium points given in section 3.

Section 5 describes two alternative characterizations of perfect equilibria a la van Damme (1991), one of them using the concept of ε-perfectness. Here it is also seen that contrary to the result for equilibrium points, there is no exact correspondence between perfect equilibria and perfect Nash equilibria of the corresponding trade-off games.

Section 6 concludes with some remarks on the case when one would apply a weaker concept of domination, providing stronger equilibria.

2 Equilibrium points for multicriteria games

We consider mixed extensions of n-person finite strategic multicriteria games. These are games with a player set $N = \{1, \ldots, n\}$ in which each player $i$ has a finite set of pure strategies $S_i = \{s_{i1}, s_{i2}, \ldots, s_{imi}\}$.

Pure strategy combinations $s \in \Pi_{i=1}^n S_i$ provide to each player $i$ “payoffs” given by an $r(i)$-vector valued function $K_i : \Pi_{j=1}^n S_j \rightarrow \mathbb{R}^{r(i)}$, i.e. player $i$ takes $r(i)$ criteria into account. Considering mixed strategies we let $A(S_i)$ represent