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ORIGINAL ARTICLE

The contribution of Very Long Baseline Interferometry to ITRF2005

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Abstract The contribution of the International VLBI Service for Geodesy and Astrometry (IVS) to the ITRF2005 (International Terrestrial Reference Frame 2005) has been computed by the IVS Analysis Coordinator's office at the Geodetic Institute of the University of Bonn, Germany. For this purpose the IVS Analysis Centres (ACs) provided datum-free normal equation matrices in Solution INdependent EXchange (SINEX) format for each 24 h observing session to be combined on a session-by-session basis by a stacking procedure. In this process, common sets of parameters, transformed to identical reference epochs and *a prioris*, and especially representative relative weights have been taken into account for each session. In order to assess the quality of the combined IVS files, Earth orientation parameters (EOPs) and scaling factors have been derived from the combined normal equation matrices. The agreement of the EOPs of the combined normal equation matrices with those of the individual ACs in terms of weighted root mean square (WRMS) is in the range of 50–60 μas for the two polar motion components and about 3 μs for UT1−UTC. External comparisons with International GNSS Serive (IGS) polar motion components is at the level of 130–170 μas and 21 μs/day for length of day (LOD). The scale of the terrestrial reference frame realized through the IVS SINEX files agrees with ITRF2000 at the level of 0.2 ppb.

Keywords Geodetic VLBI · Solution combination · Normal equations · ITRF2005

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1 Introduction

Modern global terrestrial reference frames (TRFs) form solid foundations for all kinds of Earth sciences, as well as for geodetic survey control and navigation. From the middle of the 1980s, geodetic Very Long Baseline Interferometry (VLBI) observations have contributed to the generation and maintenance of TRFs. Starting with the BIH Terrestrial System 1984 (BTS84) (Boucher and Altamimi 1985), the Bureau International de l'Heure (BIH) and its successor, the International Earth Rotation and Reference Systems Service (IERS), have been in charge of the combination of the results of the different space-geodetic techniques into one common frame, the International Terrestrial Reference Frame (ITRF) as the realisation of the International Terrestrial Reference System (ITRS) (Altamimi et al. 2002).

Up to the last ITRF realisation, which was the ITRF2000, individual Analysis Centres (ACs) were invited to submit their results directly to the ITRF Product Centre of the IERS. The inputs consisted of consolidated TRF solutions with full variance/covariance matrix from each AC. For the ITRF2005, however, only one input per technique was requested from the services of the International Association of Geodesy (IAG), i.e., from the International VLBI Service for Geodesy and Astrometry (IVS), from the International GNSS Service (IGS), from the International Laser Ranging Service (ILRS), and from the International DORIS Service (IDS). The official contribution of the IVS has been computed by the IVS Analysis Coordinator's office at the Geodetic Institute of the University of Bonn, Germany.

In the case of IVS, the IERS asked for individual data sets for each VLBI observing session of 24 h duration in Solution INdependent EXchange (SINEX) format

(Blewitt et al. 1994). SINEX files permit the transmission of the full variance/covariance information to interpret the quality of the solution to its full extent or to further combine the results with other solutions. This can be realized by reporting either the full variance/covariance matrix or the normal equation matrix of a solution setup (see the latest definition at http://www.tau.fesg.tu-muenchen.de/∼iers/web/sinex/format.php).

The latter option is mainly meant for further combinations but, if required, variance/covariance information can easily be extracted through an inversion procedure, which may have to include a datum definition if necessary. In order to facilitate combination steps by a procedure that does not require a datum definition, the IVS had decided that IVS ACs report datum-free normal equation matrices in their SINEX files to the IVS Analysis Coordinator. Consequently, the IVS input to ITRF2005 is also based on datum-free normal equations.

The generation of combined SINEX files from individual IVS analyses mainly consists of a stacking of the normal equation matrices. However, specific aspects, like a common set of parameters, identical reference epochs and *a prioris*, and especially a representative relative weighting have to be taken into account for the generation of a homogeneous and consistent combined VLBI product from the input of several IVS ACs (IVS 2005).

Since the combined data sets refer only to individual observing sessions, a consistent treatment of any site-specific modelling like (linear) drifts or episodic events are left to the ITRF Product Centre, where discontinuity information from different techniques for identical sites can be matched. Another advantage is that if the SINEX files contain Earth orientation parameters (EOPs) and station coordinates, EOPs can be determined that are consistent with the TRF. In this respect, the new approach is a first step to replace the stand-alone generation of EOPs independently of the TRF (cf. Rothacher 2000).

2 Combination of space-geodetic data

2.1 Basics of the IVS combination at the normal equation level

Combination at the level of normal equations (also known as 'adjustment of groups of observations'; cf., e.g., Mikhail 1976 or Brockmann 1997) is very close to the combination at the level of observations, if one considers a few basic requirements (e.g., identical models and identical a priori values). Thus, it can be regarded as a compromise that is much easier to realise on an operational basis than the combination at the observation level.

2.1.1 Adjustment of groups of observations

One way to solve an over-determined system of linear equations in a (weighted) least-squares sense, i.e., by minimizing $\|\mathbf{Ax}-\mathbf{b}\|_P^2$, is to compute the normal equations

$$\mathbf{Nx}=\mathbf{y} \quad \text{with} \quad \mathbf{N}=\mathbf{A}'\mathbf{PA} \ \text{ and } \ \mathbf{y}=\mathbf{A}'\mathbf{Pb} \tag{1}$$

with $\mathbf{A}$ being the design matrix and $\mathbf{b}$ being the difference vector of observed and computed observations. $\mathbf{N}=\mathbf{A}'\mathbf{PA}$ describes the 'normal equation matrix', while $\mathbf{y}=\mathbf{A}'\mathbf{Pb}$ is also known as 'the right-hand side of the normal equations'. P denotes the weight matrix of the observations Koch (1999).

Generalisation of Eq. (1) to p independent groups of observations (or even to individual observations) leads to the *addition theorem of normal equations*:

$$\left(\sum_{i=1}^{p}\alpha_i\mathbf{A}_i'\mathbf{P}_i\mathbf{A}_i\right)\hat{\mathbf{x}}=\sum_{i=1}^{p}\alpha_i\mathbf{A}_i'\mathbf{P}_i\mathbf{b}_i \tag{2}$$

with $\hat{\mathbf{x}}$ being the vector of the estimated parameters and α_i denoting weighting factors for the individual contributions (see Sect. 2.1.4). Equation (2) can be used for groups of different observation types, like GPS, VLBI, SLR and DORIS observations, as well as for groups of observations from a single technique. In general, applying this method is always possible if the individual observation groups are stochastically independent.

Considering intra-technique combinations, as done by the IVS, the question arises whether the various ACs really produce independent data sets. However, the initial observations are processed differently by the ACs in so many ways that the independence of the input normal equations may safely be assumed. The data sets differ in the number of observations included as well as in their dimensions (i.e., the number of parameters estimated).

Since every AC treats the observations in a different way (i.e., application of different stochastic models, corrections, outlier detection and elimination, etc.), normal equations of the same original set of observations differ significantly and can thus be treated as independent input to Eq. (2). A description of the different solutions used can be found in Sect. 2.2.