Multi-level arc combination with stochastic parameters

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Abstract. The method of square root information filtering and smoothing (SRIF/S) is reviewed and has been implemented in the combined square root information filter and smoother (CSRIFS) program. CSRIFS is a part of the GEOSAT space geodesy software developed at Forsvarets forskningsinstitutt (FFI, The Norwegian Defence Research Establishment). The state vectors and complete variance–covariance matrices from the analyses of a number of independent arcs of space geodesy data can be combined using CSRIFS. Four parameter levels are available and any parameter can, at each level, be represented as either a constant or a stochastic parameter (white noise, colored noise, or random walk). The batch length (i.e. the time interval between the addition of noise to the SRIF array) can be made time and parameter dependent. CSRIFS was applied in the combination of 623 very long baseline interferometry (VLBI) observing sessions. The purpose of this test was to validate the computer implementation of the SRIF/S method and to give an example of how this method can be used in the analysis of a large number of space geodetic observations. The results show that the implementation is very satisfactory.

Key words: Kalman filter – RTS smoother – SRIF/S – VLBI

1 Introduction

The analysis of global space-geodetic data is usually performed arc-wise where the data are divided into arcs or batches of typically one day (very long baseline interferometry VLBI), a few days or less (global positioning system, GPS), or one week to several years (satellite laser ranging, SLR). Each arc is analyzed individually at the observation level, where the result in principle is either a set of normal equations or a state vector and a variance–covariance matrix for each arc. In a final stage the results or a posteriori information from all arcs are combined into a global solution where the ‘global’ or ‘common’ parameters have estimates common to all arcs and the ‘arc’ parameters have individual estimates for each arc. It is possible to introduce intermediate parameter levels where the parameters have constant values for a certain pre-defined number of arcs and new values for the next same number of arcs. In this paper we will call such parameters ‘super-arc’ parameters. The terms common, super-arc and arc levels indicate a hierarchy where the arc level is the base class. A very significant reduction in computation time can be obtained with such a procedure at the cost of some reduction in precision.

It has been observed that the internal consistency of the analysis results within each individual high-precision space-geodetic technique is at the level of almost one order of magnitude better than the external consistency between the different techniques. In order to understand some of the causes of this discrepancy it is necessary to explain in general terms the analysis strategy presently being used by the analysis centers. The different types of observations are processed separately using analysis software developed specifically for each technique. The results at the arc level for a specific technique are combined into a global technique-dependent solution. With an a posteriori combination of all technique-dependent global solutions, a multi-technique global solution is established. The main problems with such a strategy are the following. (1) It is a fact that although it is said that all software packages more or less follow the International Earth Rotation Service (IERS) Conventions (McCarthy 1996), it is seen in practice that small but important differences in the implementation very often lead to coordinate differences larger than 1 cm. One example is the pole tide correction (Andersen et al. 1999). Another example is the permanent tidal deformation at the station. A general problem is that the latest IERS Convention is pretty unclear in some cases which leads to unnecessary misunderstandings and software inconsistencies. (2) It is observed that the analysis
strategy can have a significant influence on the results. This can, for example, be a problem with VLBI data when the tracking network is geometrically weak due to, for example, failure of one or more stations during the observations or for other reasons. (3) In the arc combination only specific correlations in the arc solutions are considered. A few of the combination software packages can in principle take the complete variance–covariance matrix but this is very seldom done. (4) It is a fact that each technique has systematic errors which will lead to technique inconsistencies. The applied analysis strategy cannot be used to reveal these errors. (5) The estimates of geodetic and geodynamic parameters are in general given relative to technique-dependent realizations of the reference frames. A technique-independent combined global solution will contain reference frame inconsistencies caused by, for example, technique-dependent local frame deformations which cannot be compensated for with a 7 (or 14)-parameter Helmert transformation.

Andersen (in preparation) proposes a more general procedure where the different types of data are combined at the observation level with one consistent model and one consistent strategy. The present paper presents a method where all correlations are accounted for in the process of combining the arc solutions to a global solution. There are several additional advantages with the combination of VLBI, GPS, and SLR data at the observation level. (1) One set of technique-independent estimates can be determined for the motion of a collocated station, the Earth orientation parameters, the geocenter, and the tropospheric parameters including zenith wet delay and atmospheric gradients. (2) The combination of independent and complementary information from different types of observations will reduce the parameter correlations and lead to more accurate results. The strength of VLBI is in the determination of distances and directions while the satellite techniques are especially important for the determination of the Earth’s center of mass. In order to obtain high-precision results with VLBI and GPS, the water vapor content of the troposphere or the zenith wet delay must be precisely estimated. A GPS station observes in several directions simultaneously while a VLBI station observes in one direction with a very low elevation cutoff angle. The introduction of SLR observations, independent of the water vapor, will contribute to the decorrelation of the zenith wet delay parameter from all the other estimated parameters, especially the height component of the station coordinates. (3) The proposed method will make it possible to detect and study possible technique-dependent systematic errors and biases. (4) The estimated satellite orbital elements, radio source coordinates, and nutation parameters will be realized in a long-term stable celestial reference frame realized primarily by the radio sources. GPS and SLR will contribute directly in the determination of nutation and UT1 and not only be used to estimate the rate of change of these parameters. (5) All estimates of geodetic and geodynamic parameters will in principle be given in one single realization of the terrestrial reference frame. This is especially interesting since a terrestrial reference frame realized by VLBI alone is almost free of gravity effects while reference frames realized by the satellite techniques are certainly dependent on gravity effects. (6) It is often seen that results from the analysis of VLBI data to some degree suffer from variations in the VLBI network geometry. This problem should be completely removed with the inclusion of SLR data and especially GPS data in the analyses. (7) The combined analysis of VLBI, GPS, and SLR can be used to estimate (and validate) the eccentricity vectors between the different antenna phase centers within each colocated station.

The programs presently in wide use for global geodetic analysis apply the least squares (LS) with the arc parameter elimination method (ARCPE) (Mikhail 1976), or the Kalman filter (KF) (Kalman 1960) and Rauch–Tung–Striebel (RTS) smoother (S) (Rauch et al. 1965), or the square root information filter and smoother (SRIF/S) method (Bierman 1977). All three methods consist of a forward step and a backward step.

In the forward step of ARCPE, information is collected from a series of arcs into a set of reduced normal equations where the arc parameters are eliminated. Once the global parameters are estimated the arc parameters can be sequentially found in the backward step by back substitution.

In the forward step of the KF/S and SRIF/S methods the state vector and variance–covariance matrix from each arc are treated as a new observation which is combined with the current combined filter state vector and variance–covariance matrix or the SRIF array containing information from the previous arcs. In the backward step the stochastic parameters, if any, are smoothed.

In some applications it is advantageous to have more than two parameter levels, which is the common standard amongst the arc combination software available today. One such example is the analysis of SLR data where Earth gravity coefficients, station coordinates, and velocities are typical global or common parameters. Typical arc parameters are satellite orbital elements and satellite dynamical scale factors. It is mandatory for the high-precision applications also to estimate range biases regularly with common estimates for several arcs. The range bias parameters are therefore super-arc parameters.

The arc combination software packages normally use only specific parts of the arc variance–covariance matrix, for example related to the station coordinates and velocities, the Earth orientation parameters (EOPs), the satellite orbital elements, or the radio source coordinates, even though the complete arc variance–covariance matrix is available. The complete arc variance–covariance matrix containing absolutely all estimated parameters is in most cases not used in order to save computer time and disc space. Important correlations are thus neglected. The zenith wet-delay parameter estimated in the analysis of VLBI and GPS observations is, for example, highly correlated with the vertical component of the station coordinates. Another example is the UT1 parameter, which is highly correlated with the right ascension of the ascending node (one of the satellite orbital