Linear drift and periodic variations observed in long time series of polar motion

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Abstract. Two long time series were analysed: the C01 series of the International Earth Rotation Service and the pole series obtained by re-analysis of the classical astronomical observations using the HIPPARCOS reference frame. The linear drift of the pole was determined to be 3.31 ± 0.05 milliarcseconds/year towards 76.1 ± 0.80° west longitude. For the least-squares fit the a priori correlations between simultaneous pole coordinates x_p, y_p were taken into account, and the weighting function was calculated by estimating empirical variance components. The decadal variations of the pole path were investigated by Fourier and wavelet analysis. Using sliding windows, the periods and amplitudes of the Chandler wobble and annual wobble were determined. Typical periods in the variable Chandler wobble and annual wobble parameters were obtained from wavelet analyses.

Key words: Polar motion – Chandler wobble – Wavelet analysis

1 Introduction

Polar motion research still has a lot of unanswered questions:

Is there a long-term drift of the pole with respect to the Earth’s surface?
If so, what are the main causes for this drift, which is often called ‘secular polar motion’?
What are the causes of the observed decadal variations of polar motion?
Are the decadal variations stable or irregular?
Why is the amplitude of the Chandler wobble (CW) not steadily decreasing with time due to damping, and what are its excitation mechanisms?

What are the reasons for the apparent rapid variations of the CW period, also expressed as phase jumps and/or the strong CW amplitude variations, which have been reported by many from analysis of polar motion time series?

The reasons for the investigation presented in the present paper are as follows.

(1) New precise and consistent time series of polar motion exist, as will be described in the next paragraph.

(2) The wavelet analysis is a relatively new and useful technique to detect quasi-periodic, partly irregular variations in time series.

(3) Powerful computers are available which allow big matrices to be inverted in a short time. Thus, the stochastical model of the analyses can be extended and completed, as will be shown.

Starting from these prerequisites and using the tools mentioned above, an analysis was carried out to determine the linear drift and decadal variations of the pole and to investigate the CW and the annual wobble (AW). Wavelet transformation was used, which has been proven a powerful tool for investigating the time-variable Earth rotation (see e.g. Chao and Naito 1995; Gibert et al. 1998 and references therein). The goal of this investigation is to fully describe what can be seen in the polar motion data and to reveal previously unstudied effects. Further interpretations of the results might help to answer the questions given above.

2 Long time series of polar motion

Several time series of polar motion were initially analysed with respect to a linear drift. The linear model was then combined with periodical models of CW and AW. The decadal variations were investigated by Fourier analysis and wavelet analysis. Finally, the CW and AW parameters were repeatedly determined by a sliding window analysis, and their variability was analysed by wavelet transformation.
The following long time series of polar motion were analysed.

(1a) The C01 series (1861.0–1997.0) published by the International Earth Rotation Service (IERS), here referred to as IERS C01 (1861.0–1997.0).

(1b) As regular astronomical observations by the International Latitude Service (ILS) started in 1899, we also looked at a truncated version of the above for the time interval from 1899.7 to 1992.0. This series will be called IERS C01 (1899.7–1992.0).

(2a) The pole series OA97 (1899.7–1992.0) obtained by re-analysis of optical astrometry observations referred to the HIPPARCOS catalogue as described by Vondrak (1999a). This series, based on 4.3 million observations, will be used for the determination of the linear drift, for the sliding window analyses and the wavelet analyses.

(2b) We also used a new time series recently received from Vondrak (Vondrak and Ron 2000) called OA99 (1899.7–1992.0), based on 4.5 million observations. This is more observations than in the OA97 series, although those from the Ukieh station since 1960 were neglected by Vondrak and Ron (2000) because of probable local or regional movements of that station. The OA99 series also refers to a slightly different reference frame due to proper motion corrections.

3 Linear drift and decadal variations of polar motion

3.1 Model

Our numerical model contains a purely linear part for ‘secular polar motion’ (offset and drift parameters a, b, c, d) and two periodical elliptical motions representing CW (parameters $R_{1a}$, $R_{1b}$, $\omega_{1a}$, $\phi_{1a}$, $\phi_{1b}$) and AW (parameters $R_{2a}$, $R_{2b}$, $\omega_{2a}$, $\phi_{2a}$, $\phi_{2b}$):

\[ x_p = a \cdot t + b + R_{1a} \cos(\phi_{1a} + \omega_{1a} t) + R_{2a} \cos(\phi_{2a} + \omega_{2a} t) \]  

\[ y_p = c \cdot t + d + R_{1b} \sin(\phi_{1b} + \omega_{1b} t) + R_{2b} \sin(\phi_{2b} + \omega_{2b} t) \]  

The semi-major and semi-minor axes of an elliptical motion correspond to the amplitude of a circular motion. If an elliptical motion of the pole for a specific frequency is detected this could allow this variation to be referred to the geographical distribution of particular geophysical fluids. All parameters were estimated simultaneously by a least-squares (LS) fit as described in the following sections.

3.2 Choice of weighting function

One important issue in LS parameter estimation is the right choice of the weight matrix $P$, i.e. the determination of an appropriate weighting function. Without considering the influence of the geometry on the results of the LS fit, the weights $p_i$ were chosen according to three different weighting functions in our study:

(1) $p_i = 1$, which means equal weights. This simple method was used in most previous analyses [see the comprehensive tables given in McCarthy and Luzum (1996), Gross and Vondrak (1999) and Korsun and Yatskiv (1999)]. However, this approach does not take into account the fact that the precision of the measurements has increased considerably over the century.

(2) $p_i = \text{const}/\sigma_i^2$, with $\sigma_i^2$ being the variances of the ‘observed’ pole coordinates as given in the time series. When using this weighting function, which is usually recommended in textbooks on LS analysis, care has to be taken that a small number of observations do not influence and perhaps bias the results, whereas the other observations — although formally entered into the LS fit — are almost neglected (e.g. a difference in $\sigma_i$ by a factor of 10 will cause differences of the weights by a factor of 100).

(3) $p_i = \text{const}/(\sigma_i^2 + \sigma_0^2)$, with $\sigma_0^2$ being a constant. This approach allows us to take into account the different precisions of the data but not as much as the formal errors indicate. A reasonable value for $\sigma_0^2$ can be obtained by a $\chi^2$ test when comparing the (a posteriori) variance with the a priori variance of the observables. This procedure is called estimation of empirical variance components.

Figure 1a shows the three weighting functions for series OA97 depending on the a priori formal errors $\sigma_i$. The constant $\sigma_0^2$ was determined by a $\chi^2$ test and set to 0.054 arcseconds$^2$.

3.3 A priori correlations between simultaneous pole coordinates $x_p$, $y_p$

In both time series OA97 and OA99 calculated by Vondrak (1999a), and by Vondrak and Ron (2000), correlation coefficients are given between simultaneous pole coordinates $x_p(t_i)$, $y_p(t_i)$ (Fig. 1b). They are usually between 0.2 and 0.5; some of them reach 0.9 or more. It is interesting to note that with the increasing precision and intensity of astronomical measurements since late 1950s, the correlations between $x_p$ and $y_p$ have decreased and seldom exceed ±0.4. The correlations also depend on the geometry of the network, i.e. the geographic distribution of the observations. Two solutions were carried out, first the ‘uncorrelated’ (standard) solution neglecting the correlations, and second a solution in which the given correlations between the observables were considered in the LS fit (‘correlated approach’). In the latter approach the block-diagonal variance–covariance matrix has to be calculated. The inversion of this matrix requires more central processing unit (CPU) time than that of a diagonal matrix, but poses little problem.