The modified wave estimator as an alternative to a Kalman filter for real-time GPS/GLONASS–INS integration

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Abstract. The performance of a Kalman filter is essentially limited by the description of the input noise and therefore it is difficult to improve the estimation procedure within the framework of traditional estimation theory. One way to further improve performance is to describe the system in a deterministic sense for a meaningful, but short duration of time. A method called the modified wave estimator (MWE) is used as an alternative to a conventional Kalman filter, where the non-white disturbances are modeled using simple curves or waves, rather than shaping filters driven by known input noise values. The major advantages of this method compared to a conventional Kalman filter are that the estimation accuracy is higher, especially for comparatively weak observables, and is less sensitive to the description of input noise. Results from an integrated global positioning system (GPS)/GLONASS (Global Navigation Satellite System) – inertial navigation system (INS) test are used to demonstrate the performance accuracy of the system using both a Kalman filter and the MWE approach. Results and their analyses are presented with emphasis on situations where improved estimation can be achieved using this new technique.

Key words. Modified wave estimator · Kalman filter · Optimal estimator · GPS/GLONASS–INS · Accuracy

1 Introduction

Kalman filtering has gained widespread use in applied estimation and control due to its desirable characteristics, such as minimization of average mean square errors (MSE), and ease of recursion and implementation (Brown and Hwang 1992). It renders optimal estimation in such cases as a minimum mean square error (MMSE) estimate, a maximum likelihood estimate (MLE), or a maximum a posteriori (MAP) estimate (Gelb 1979; Maybeck 1994). However, it is optimal only if the input and measurement noise values are known accurately and, in addition, the estimation quality is only as good as the underlying model. Furthermore, in a Kalman filter, the weakly observed states require a longer time to converge and thus estimates during this period yield poorer results (Salychev 1995). At steady state the estimation accuracy is limited by the input noise.

Non-conventional approaches proposed by Salychev (1995, 1998), i.e. scalar and wave estimation techniques, are precursors to the algorithm presented below. Scalar estimation is not highly sensitive to the accuracy of the mathematical model and input noise statistics, and allows each of the state variables to be estimated separately. In wave estimation, input disturbances are described by pseudo-deterministic models which are valid over a short time interval. The method described in this paper is related to further developments and enhancements of the wave estimation technique.

Kalman filters have been applied to global positioning system–inertial navigation system GPS–INS integration in many cases over the past decade. The advantages of integrating GPS with INS are discussed by Cox (1980) in terms of improved solution accuracy, aided acquisition and tracking, as well as adaptive tracking. Several authors have described various ways to integrate GPS with INS. For example, Callender (1989) integrated a Ferranti FIN 1041 INS with a P-code receiver using a Kalman filter for airborne applications, while Diesel (1988) described an INS-aided GPS integrity monitoring system. Eissfeller and Spietz (1989) examined the accuracy potential of Honeywell’s strapdown laser inertial system to aid kinematic GPS using feed-forward and feedback Kalman filters, while Cannon (1992) achieved accuracies of 5 cm for an integrated road positioning system. Lapucha et al. (1990) developed a highway survey system using a ring-laser gyro INS integrated with a differential carrier-phase GPS
system, while Wei and Schwarz (1990) devised different decentralized Kalman filter configurations which are flexible to integrate GPS, INS and other sensors. In all cases, Kalman filters of various forms were used. Fundamentals of INS can be found in Britting (1971) on a comprehensive review of GPS and inertial integration is available in Greenspan (1996).

In this paper, the concept of a modified wave estimator (MWE) is presented and compared to a conventional Kalman filter in order to evaluate the accuracy performance of a GPS/GLONASS–INS (Global Navigation Satellite System) system using both estimation techniques.

2 Modified wave estimation

The MWE technique was described by Salychev (1995) and is based on the principle that input disturbances can be described by deterministic means for short time periods. Instead of using a shaping filter driven by white Gaussian noise, the MWE models input disturbances as known base functions with unknown intensities which can be estimated (see Figs. 1 and 2).

A similar technique is described by Lichten (1990) with regard to the GPS inferred positioning system (GIPSY), a multi-satellite batch sequential pseudo-epoch state process noise filter for estimation of GPS satellite orbits and other parameters. In this method, the filter divides the measurements into finite time intervals, known as batches, during which all the process noise parameters are assumed to be piecewise constant. After filtering is complete, a smoother works recursively backwards in time to optimally update the computed estimates and covariances. Another method that is related to MWE is the Schmidt–Kalman filter (Brown and Hwang 1992) wherein only the desired states are estimated, taking into account the influence of other non-estimated states.

In MWE, the input disturbance can be represented as follows:

\[ w(t) = c_1 f_1(t) + c_2 f_2(t) + \cdots + c_n f_n(t) \]  

where \( w(t) \) is the input disturbance, \( f_1(t) \ldots f_n(t) \) are known base functions, e.g. horizontal lines, inclined lines, exponential functions and \( c_1 \ldots c_n \) are unknown coefficients which vary from one instant in time to the next.

The system is represented as

\[ x_k = \Phi_{k-1} x_{k-1} + \delta_{k-1} \]  

and the measurement model assuming a stationary process is given as

\[ z_k = H x_k + v_k \]  

where

- \( x_k \) is the total state vector
- \( \Phi_{k-1} \) state transition matrix
- \( \delta_{k-1} \) consists of impulse functions appearing once every \( N \) time steps. Their intensities are the same as the unknown coefficients of the wave description
- \( z_k \) measurement vector
- \( H \) observation matrix
- \( v_k \) white Gaussian measurement noise with zero mean and known covariance \( R_k \).

Using Eqs. (1)–(3), it is possible to describe the system in a deterministic sense during a short time interval \( NT \), where \( T \) is the sampling period and \( NT \) is called the cycle time. The main issue is then the selection of an appropriate cycle time. A small cycle time allows a more accurate representation of the system, but it may not be sufficient for all the state vectors to converge. On the other hand, a large cycle time ensures convergence, but may degrade the estimation accuracy. A more prudent approach is to segregate the state variables into two groups based on their degree of observability. The observability condition is defined as the ability to determine the state variables from the given measurements (Gelb 1979). In this approach, the first group is comprised of all the strongly observed states, normally the basic system model, and the second part comprises all the weakly observed states, normally the state variables for wave representation. It is then possible to have a shorter wave cycle ensuring convergence of only the strongly observed states. Weakly observed states, even though not converged during the cycle time, can be estimated separately in a second group.

Through separation, the basic system model has two components in a wave cycle. The first component is the influence of the strongly observed states on themselves and the second component is the influence of the weakly observed states on the strongly observed states. There-