Quasi-stationary sea surface topography estimation by the multiple input/output method

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Abstract. Multiple input/multiple output system theory (MIMOST) is briefly presented, and the application of the method to the quasi-stationary sea surface topography (QSST) estimation and the filtering of the input observations are discussed. The repeat character of satellite altimetry missions provides more than one sample of the measured sea surface height (SSH) field, and an approximation of the input signal and error power spectral densities can be determined using this successive information. A case study in the Labrador Sea is considered using SSHs from ERS1 phases C and G, ERS1-GM, ERS2 phase A and TOPEX/POSEIDON altimetric missions in combination with shipborne gravity anomalies. The time period of the observations in this study is from 1993 to 1998. Some comparisons between the techniques used for the power spectral density approximation are carried out and some remarks on the properties of the estimated QSST are presented.

Key words: Multiple Input/Multiple Output Method – Satellite Altimetry – Sea Surface Topography – Geoid – Gravity

1 Introduction

The use of spectral methods in physical geodesy has been developed during recent decades. The basic advantage of the analysis in the spectral domain is the algebraic simplicity of the convolution integrals. As is well known, the convolution integrals are transformed to multiplications in the spectral domain and the evaluation of some complicated formulas in gravity field modeling is speeded up.

The heterogeneous data combination and the appropriate error propagation using spectral methods are presented in Sideris (1996). The input/output system theory (IOST) is based on the analysis given by Bendat and Piersol (1986) and the proper adaptation in gravity field applications is discussed in Sideris (1996). Gravity field applications of IOST can be found in Li (1996) and Tziavos et al. (1996a, b, c, 1998a) and an application to airborne gravimetry was presented by Wu and Sideris (1995). The similarities and the differences between IOST and least-squares (LS) collocation were analyzed by Sansò and Sideris (1997). The multiple input/multiple output system theory (MIMOST) has recently been used by Andritsanos and Tziavos (1999) and Andritsanos et al. (1999). In these papers, some simulation studies were performed in the Labrador Sea using Gaussian noises for the input gravity and geoid data filtering, as well as for the quasi-stationary sea surface topography (QSST) estimation.

Many researchers developed appropriate algorithms for an accurate estimation of the sea surface topography (SST). Engels (1983) presented a global solution based on harmonic analysis using SEASAT altimeter data. A comparable solution was presented by Tai and Wunsch (1984) using filtered SEASAT data to reduce aliasing. A global solution based on harmonic analysis of pure oceanographic data from Levitus (1982) was computed by Engels (1987b). In another approach, a simultaneous radial orbit error reduction was achieved (Engels, 1987). SEASAT altimeter data were also used in the SST estimation presented by Engels and Knudsen (1989). Knudsen (1991) estimated the QSST in the Faeroe Islands, as well as the time-variant part with (LS) collocation, and constructed error covariances using information from previous studies and Butterworth filters for the analytical expressions. Hwang (1995) estimated a global SST solution based on orthonormal functions for GEOSAT. Rapp et al. (1996) recommended the use of spherical harmonics for SST representation, followed by transformation to the orthonormal basis. Sanchez et al. (1997) implemented the height function representation, introduced by Rao et al. (1987), and compared it with spherical harmonics. Recently, Pavlis et al. (1998) presented a global estimation of the SST based on spherical harmonics and an eigenvalue analysis of the Proudman functions.
In the present paper, a QSST estimation procedure based on MIMOST is presented. A 6 year time period is chosen in order to verify the quasi-stationary character of the approximation. Precomputed input error and signal information are used for the sea surface height (SSH) observations based on a successive track analysis. Two slightly different methods are applied for determining the input error power spectral density (PSD) and the resulting two-dimensional (2-D) covariances are compared with each other. The PSDs/covariances are calculated for each separate year and each specific satellite, and are introduced into the MIMOST procedure for the QSST estimation.

2 Theoretical background

2.1 MIMOS theory

A multiple input/multiple output system (MIMOS) is presented in Fig. 1, where \( y_i \) are the input observations, \( y_i \) are the pure input signals, \( m_i \) are the input noises, \( h_i \) are the unknown transfer functions that filter out the input noise, \( x_i \) are the unknown output signals, and \( e_j \) are the output noises. The total number of input data \( i \) may be equal to or different than the number of output signals \( j \). The interaction between every input and output is also presented in Fig. 1.

The aim of the method is the determination of the impulse response functions \( h_i \) based on some criterion. The minimization criterion of the output error PSD is used for the transfer function estimation; see, for example, Bendat and Piersol (1986) and Sideris (1996). If matrix notations are applied for the description of the system depicted in Fig. 1, then the following equations are valid; see, for example, Bendat and Piersol (1986), Sideris (1996), Andritsanos and Tziavos (1999), and Andritsanos et al. 1999:

\[
\begin{align*}
Y_o &= \begin{bmatrix} Y_1 + M_1 \\ Y_2 + M_2 \\ \vdots \\ Y_q + M_q \end{bmatrix}, \\
X_o &= \begin{bmatrix} X_1 - E_1 \\ X_2 - E_2 \\ \vdots \\ X_w - E_w \end{bmatrix}
\end{align*}
\]

where capital letters stand for the spectra of the respective quantities and the output error matrix is given as follows:

\[
\begin{align*}
E &= X - X_o = X - H_{yo}^T HY_o
\end{align*}
\]

The transfer function matrix is of the form:

\[
H_{yo} = \begin{bmatrix} H_{y1x1} & H_{y1x2} & \cdots & H_{y1xn} \\ H_{y2x1} & H_{y2x2} & \cdots & H_{y2xn} \\ \vdots & \vdots & \ddots & \vdots \\ H_{ynx1} & H_{ynx2} & \cdots & H_{ynxn} \end{bmatrix}
\]

which shows the dependence of each output on the inputs. If Eq. (2) is multiplied by its complex conjugate form, an estimation of the output error PSD can be computed following the periodogram approach as described in Marple (1987) and Bendat and Piersol (1986)

\[
P_{ee} = P_{xx} - H_{yo}^T P_{yy} H_{yo} + H_{yo}^T P_{yx} H_{yo}^T
\]

where the asterisk stands for the complex conjugate of the matrix elements. The optimal transfer functions can be calculated by the minimization of the output error PSD matrix \( P_{ee} \) as follows:

\[
\begin{align*}
\frac{\partial P_{ee}}{\partial H^*_{yo}} &= 0 \\
\Rightarrow -P_{yx} + H_{yo}^T P_{yy} H_{yo} &= 0
\end{align*}
\]

Assuming no correlation between input signals and input noises, the optimal transfer function matrix is

\[
H_{yo} = P_{xx}^{-1} P_{yx} = P_{yx} (P_{yy} + P_{mm})^{-1}
\]

Then the output signal vector and the output noise PSD matrices are

\[
X_o = H_{yo} Y_o = P_{xy} (P_{yy} + P_{mm})^{-1} (Y + M)
\]

The similarities between MIMOST and least-squares collocation (LSC), as presented in Sanso and Sideris (1997), can be observed in Eq. (8). Direct comparison between Eq. (8) and the classical solution of LSC presented in Moritz (1980) justifies this statement.

When both input and output are known, the method is focused on the optimal estimation of the transfer function between the input and the output signals (Bendat and Piersol, 1986). In gravity-field-related applications, the output signals are unknown. The fundamental difficulty of the method in the current application is the estimation of the input–output PSD, when the output signal is unknown. In this specific case, the evaluation of the input–output PSD matrix is possible only if the input noise PSD matrix is known. Then, \( P_{xy} \) can be computed by

\[
P_{xy} = H_{xy} P_{yy} = H_{xy} (P_{yyx} - P_{mm})
\]

where \( H_{xy} \) is the transfer function matrix which connects the pure input and output signals. For example, if gravity anomalies are chosen as the input signals, and the geoid as the output signal, then, theoretically, \( H_{xy} \) is nothing other than the Stokes operator in the frequency domain. Using Eq. (9), the final solution of Eq. (8) is given by