Comparison of reliability and geometrical strength criteria in geodetic networks

A. A. Seemkooei

Department of Surveying Engineering, Faculty of Engineering, The University of Isfahan, Isfahan, 81744, Iran
E-mail: amiri@eng.ui.ac.ir/ar_amiri@yahoo.com; Tel.: +98-311-7932680; Fax: +98-311-682887

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Abstract. The proper and optimal design and subsequent assessment of geodetic networks is an integral part of most surveying engineering projects. Optimization and design are carried out before the measurements are actually made. A geodetic network is designed and optimized in terms of high reliability and the results are compared with those obtained by the robustness analysis technique. The purpose of an optimal design is to solve for both the network configuration (first-order design) and observations accuracy (second-order design) in order to meet the desired criteria. For this purpose, an analytical method is presented for performing the first-order design, second-order design, and/or the combined design. In order to evaluate the geometrical strength of a geodetic network, the results of robustness analysis are displayed in terms of robustness in rotation, robustness in shear, and robustness in scale. Results showed that the robustness parameters were affected by redundancy numbers. The largest robustness parameters were due to the observations with minimum redundancy numbers.

Key words: First-order Design – Second-order Design – Combined Design – Reliability Criterion – Robustness Analysis

1 Introduction

This section deals with the types of problems most widely encountered in the design of a geodetic network. Optimization means minimizing or maximizing an objective function which represents the criteria adopted to define the 'quality of a network'. Generally, the quality of a control geodetic network is characterized by its precision, reliability and strength, and economy. The reliability of a geodetic network, which was proposed by Baarda (1968), can be understood as the ability of the network to detect gross errors in the observations and to be resistant against undetectable errors. The geometrical strength analysis (robustness analysis) of a geodetic network which was proposed by Vaniček et al. (1990) deals with the strain and is another aspect of reliability criteria. The main purpose of the present paper is to design and optimize a geodetic network in the sense of high reliability and compare the results with the robustness analysis. Different optimization problems are usually classified into different orders; Grafarend (1974) identifies four orders of design:

(a) zero-order design (ZOD): design of a reference system
(b) first-order design (FOD): design of the network configuration
(c) second-order design (SOD): selection of the weights for the network observations
(d) third-order design (THOD): addition of observations to improve an existing network.

When the FOD and SOD design problems are solved simultaneously, it is called a combined design (COMD) problem (Vaniček and Krajewsky 1986).

There are two methods that can be used to solve the design problems, namely, analytical method and trial and error method. In the trial and error method, a solution to the design problem is postulated upon which the design criteria are computed. Should either of these criteria be not fulfilled, a new solution (usually by slightly altering the original postulate) is postulated and the criteria functions are recomputed. The procedure is repeated until a satisfactory network is found. For more information, the interested reader is referred to Cross (1985). In contrast, the so-called analytical method offer specific algorithms for the solution of particular design problems which do not require human intervention. The term analytical design is used to describe a method that solves a particular design problem by a unique series of mathematical steps. In fact, such an algorithm will automatically produce a network that will satisfy the user quality requirements and will be optimum in some mathematical sense.
Kuang (1991) developed an effective analytical method for optimization and design of deformation monitoring schemes which can be used for FOD, SOD, and/or COMD. In this method, the Taylor series expansion is introduced to linearize non-linear matrix functions related to the design criteria of the deformation monitoring network. Then, it is possible to perform either separate (e.g. FOD or SOD) or simultaneous (e.g. COMD) fully analytical optimal solution of the network configuration and the observational plan using the methodology of operations research (e.g. linear programming). This method was used in the optimization and design of geodetic networks in the sense of high reliability by Amiri Seerkooei (1998). More details are presented in Sect. 4. The main objectives of the present paper are as follows:

(a) to define the optimality criteria for the reliability and geometrical strength of a geodetic network;
(b) to formulate optimization mathematical models in the sense of high reliability for FOD, SOD, and/or COMD;
(c) to optimize a simulated geodetic network in the sense of high reliability and compare the results with robustness analysis (geometrical strength analysis).

2 Measures and criteria for reliability

Generally, the reliability of a network can be understood as the ability of the network to detect and resist against gross errors in the observations. In this respect, the internal reliability and external reliability are usually distinguished. The former refers to the ability of the network to allow for the detection of blunders by hypothesis testing, while the latter is related to the effect of the undetected gross errors on the estimated parameters. Reliability of a network depends on the geometry of the network and the accuracy of the observations. At the design stage, we look for an optimal network in the sense of high reliability and strength, which minimizes the magnitude of undetectable gross errors in the observations, and consequently minimizes the effects of the undetected errors on the estimated parameters.

As mentioned above, internal reliability is a measure of a network quality in detecting outliers in observations by the one-dimensional outlier test. It refers to the lower bounds of detectable gross errors which can be derived by the following equation (Baarda 1968):

\[ \nabla_{0,i} l_i = \frac{\delta_0 \sigma_i}{\sqrt{r_i}} \]  

(1)

where \( \delta_0 \) is the lower bound for the non-centrality parameter and is a function of type I and II errors, and \( \sigma_i \) and \( r_i \) are the standard deviation and redundancy number of the \( i \)th observation, respectively. The redundancy numbers of the observations are the diagonal elements of the matrix \( R \)

\[ R = I - A(A^T P A)^{-1} A^T P \]  

(2)

where \( I \) is the identity matrix, \( A \) is the design (configuration) matrix, and \( P \) is the weight matrix of the observations.

Another reliability measure that Baarda developed in 1968 is called the external reliability. It refers to the maximum effect of the undetectable gross error (\( \nabla_{0,i} l_i \)) on the estimates of unknown parameters. The influence of the maximum undetectable error \( \nabla_{0,i} l_i \) on coordinates (i.e. \( \nabla_{0,i} \hat{x} \)) is given by

\[ \nabla_{0,i} \hat{x} = (A^T P A)^{-1} A^T P \nabla_{0,i} l_i \]  

(3)

From the above discussion, it can be seen that a geodetic network should be designed such that:

(a) gross errors should be detected and eliminated as completely as possible. An undetectable gross error in an observation should be small in comparison with its standard deviation; and
(b) the effect of an undetected error on the coordinates should be as small as possible.

From Eqs. (1) and (3) we can see that the larger the redundancy number \( r_i \), the smaller the size of the undetectable gross error as well as its influence on the estimated coordinates. Baarda (1968) argues that it is desirable to have an approximately constant value for all \( r_i \) so that the ability of detecting gross errors will be the same in every part of the network. With this in mind, a special reliability criterion can be of the type

\[ \min(r_i) \to \max \]  

(4)

This is known as the general criterion for the internal and external reliability.

3 Robustness analysis

The effect of errors on the network is better handled as a virtual deformation and thus depicted by a more appropriate technique than the external reliability. This method, which is known as ‘robustness analysis’, is based on the concept of strain. The use of strain to analyze the strength of a geodetic network was first proposed by Vaniček et al. (1981), later developed by Dare (1983), and finally culminated in 1990 with Vaniček et al. (1990).

Strain is a purely geometric approach to the analysis of the deformation of a physical body. It is defined as the rate of change (i.e. gradient) of an object's displacement field with respect to position. Given a two-dimensional displacement field, \( u(x,y) = (u,v)^T \), as a function of position \( x = (x,y)^T \), the strain matrix \( E \) consists of four linear displacement gradients given by (Vaniček et al. 1990)

\[ E = \nabla(u) = \frac{\partial u(x,y)}{\partial x} = \begin{pmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \end{pmatrix} = \begin{pmatrix} e_{ux} & e_{uy} \\ e_{ux} & e_{vy} \end{pmatrix} \]  

(5)

where the derivatives are evaluated at the point of interest. It contains all of the strain information about the displacement field. However, it cannot be easily