Recursive equilibrium in endogenous growth models with incomplete markets

Abstract This paper analyzes the existence of recursive equilibria in a class of convex growth models with incomplete markets. Households have identical CRRA-preferences, production displays constant returns to scale with respect to physical and human capital, and all markets are competitive. There are aggregate productivity shocks that affect aggregate returns to physical and human capital investment (stock returns and wages), and there are idiosyncratic shocks to human capital (idiosyncratic depreciation shocks) that only affect individual human capital returns. Aggregate and idiosyncratic shocks follow a joint Markov process. Conditional on the aggregate state, idiosyncratic shocks are independently distributed over time and identically distributed across households. Finally, households have the opportunity to trade assets in zero net supply with payoffs that depend on the aggregate shock, but markets are incomplete in the sense that there are no assets with payoffs depending on idiosyncratic shocks. It is shown that there exists a recursive equilibrium for which equilibrium prices (returns) only depend on the exogenous aggregate shock variable (the wealth distribution is not a relevant state variable). Moreover, the allocation associated with this recursive equilibrium is identical to the equilibrium allocation of an economy in which households live in autarky and face both aggregate and idiosyncratic risk.

Keywords Recursive equilibrium · Incomplete markets · Endogenous growth

JEL Classification Numbers D52 · G50

I would like to thank for helpful comments Peter Howitt, Bob Lucas, Michael Magill, Tomo Nakajima, Herakles Polemarchakis, Martine Quinzii, Kevin Reffett, an anonymous referee, and seminar participants at various universities and conferences.

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1 Introduction

Recent work on dynamic general equilibrium models with infinitely-lived agents and uninsurable idiosyncratic risk has provided important insights into the macroeconomic effects of market incompleteness.\(^1\) One drawback of this incomplete-markets approach to macroeconomics is that recursive equilibria are in general difficult to compute even for simple economic environments.\(^2\) A second shortcoming is the lack of a general proof of existence of recursive equilibria for standard economies.\(^3\) Both these “tractability problems” are closely related to the fact that the endogenous wealth distribution is in general a relevant state variable. This paper presents a macroeconomic model with incomplete markets that is highly tractable in the sense that there are always recursive equilibria for which endogenous asset prices and rental rates only depend on exogenous shocks. This simplicity of equilibrium means that issues of existence and comparative dynamics can be studied at a level of generality comparable to the complete-markets literature, and that many quantitative applications are computationally straightforward.

The model is an incomplete-markets version of the class of convex growth models analyzed by, among others, Alvarez and Stokey (1998), Caballe and Santos (1993), Jones and Manuelli (1990), King and Rebelo (1990), Ortigueira (2000), and Rebelo (1991).\(^4\) More specifically, households have identical CRRA-preferences, production displays constant returns to scale with respect to reproducible input factors, and all markets are competitive. For the sake of concreteness, this paper considers the case of two input factors, namely physical and human capital. There are aggregate productivity shocks that affect aggregate returns to physical and human capital investment (stock returns and wages), and there are idiosyncratic human capital shocks (depreciations shocks) that only affect individual human capital returns. Aggregate and idiosyncratic shocks follow a joint Markov process. Conditional on the aggregate state, idiosyncratic shocks are independently distributed over time and identically distributed across households. Finally, the financial market structure is incomplete in the sense that there are no assets with payoffs that depend on idiosyncratic shocks. However, households have the opportunity to trade stocks (accumulate physical capital) and any asset (bond) in zero net supply with payoffs that depend on the aggregate shock variable. In particular, all households can borrow and lend at the common risk-free rate. Moreover, households’ ability to trade existing assets is only limited by their ability to repay their debt in the future. In short, the only market imperfection is the lack of explicit insurance markets for idiosyncratic human capital risk.

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\(^1\) See, for example, Langqvist and Sargent (2000) for a recent survey.

\(^2\) For applied work relying on computational methods, see, for example, Aiyagari (1994); Den Haan (1997); Heaton and Lucas (1996); Huggett (1993); Krusell and Smith (1998), and Rios-Rull (1996).

\(^3\) Duffie et al. (1994) show the existence of stationary recursive (Markov) equilibria for exchange economies, but they rule out short-selling (borrowing) by assumption and use a state space that includes endogenous variables in addition to the wealth distribution. Becker and Zilcha (1997), Datta et al. (2004), and Miao (2004) extend the analysis to production economies, but again rule out short-selling by assumption. Moreover, Miao (2004) uses a state space that includes endogenous variables in addition to the wealth distribution and Becker and Zilcha (1997) confine attention to economies with no idiosyncratic risk.