Effort maximization in asymmetric contest games with heterogeneous contestants

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Abstract Contest rules are set up by administrators who frequently have discretionary power in specifying the details of these rules, i.e., they can bias the contest rules toward specific contestants in order to further their prime objective. We derive the optimal bias of the contest rule for a contest administrator, who is interested in maximizing the total efforts expended in the contest. The solution is obtained in closed form for a widely used class of n-person contest games. Setting the optimal bias has important implications: (i) there is never exclusion of strong players, instead there is (endogenously induced) inclusion of weak contestants; (ii) the contest administrator will optimally level the playing field by encouraging weak contestants, but he will not equalize the contestants’ chances unless they are identical; and (iii) at least

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three contestants will be active in equilibrium of the optimal contest, irrespective of heterogeneity.

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1 Introduction

Competitive situations like rent-seeking, lobbying, labor and promotion tournaments, etc. are frequently modeled as contest games where agents compete for a fixed prize by exerting nonrefundable effort. In some of these situations, the contest administrator has substantial discretionary power in specifying the exact rules of the contest, for instance, by favoring or handicapping specific contestants, cf. Tsoulohas et al. (2007) for labor tournaments, and Epstein et al. (2011) for public procurement and lobbying contests. The resulting biased contest rule affects the contestants’ incentives to exert effort, which gives the contest administrator some control over the contest outcome. Hence, by specifying the optimal bias, the contest administrator can affect the exerted effort levels of the contestants. From the perspective of the administrator who seeks to maximize total exerted effort, the question is then how the contest rule should be biased to achieve this objective. Should the administrator deviate from a neutral contest rule by encouraging weak contestants to compete in the contest by exerting higher effort? Should he bias the contest rule to such an extent that strong contestants are discouraged from participation in order to increase competitive pressure between remaining contestants? Our model is intended to address these questions by determining the optimal bias of the contest rule from the perspective of the respective contest organizer.

We are going to model the described situation as a standard asymmetric lottery contest game with \( n \) heterogeneous contestants under complete information in the style of Tullock (1980), cf. Congleton et al. (2008). Here, the administrator grants a prize to the winning contestant where the winning probabilities depend on the contestants’ relative exerted efforts. Moreover, the impact of exerted effort on the contest outcome can be biased by the contest administrator through his choice of the contest success function. This simple asymmetric lottery contest rule is widely used in the applied literature and has the advantage of being sufficiently tractable for our analysis; its use can be justified on several grounds. It is, for instance, a specific case of an asymmetric contest success function axiomatized in Clark and Riis (1998), for which also micro-foundations exists, cf. Corchón and Dahm (2010) and Jia (2008). The seminal reference for the axiomatic characterization of symmetric contest success functions is Skaperdas (1995).

Moreover, our complete information setup lends itself directly to a competitive market interpretation: consider an employer who can distribute a fixed wage budget (normalized to value 1) to \( n \) agents in order to incentivize them to exert effort in output production, i.e., agents have to “pay” for wages with output that they have produced. Suppose \( n \) agents supply \((x^1, \ldots, x^n)\) units of output, then market clearing requires