Saving behavior in stationary equilibrium with random discounting*

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Summary. We study the implications of random discount rates of future generations for saving behavior and capital holdings in a steady state competitive equilibrium with heterogeneous population. A well-known difficulty in deterministic economies with heterogeneous households is that in steady state only the most patient households hold capital. In this paper we state conditions under which this random discounting is sufficient for households other than the most patient ones to save. We thus provide a simple and natural way of overcoming the aforementioned difficulty.

Keywords and Phrases: Dynamic equilibrium with heterogeneous households, Random discounting, Saving.

JEL Classification Numbers: E13, E20.

1 Introduction

The analysis of infinite-horizon, deterministic, dynamic models in which individuals face borrowing constraints shows that in steady-state equilibrium only the most patient households hold capital. All other households have wages as their only source of income (Becker, 1980). This conclusion is contradicted by the most casual observations. To overcome this difficulty Becker and Zilcha (1997) analyze the saving behavior and the ownership of capital in an economy with

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stochastic aggregate production. The random shocks to aggregate output imply
that interest rate, the wage rate and, consequently, income are random variables.
They show that, in stationary equilibrium, precautionary savings induces capital
holdings by households other than the most patient.

In the present paper we examine the steady-state saving behavior and the
ownership of capital when the rates of discount implicit in the preferences of
future generations are random. We assume that these idiosyncratic variations in
tastes average out in the aggregate so that the aggregate capital stock, output,
wages and interest rates are nonrandom. We show that, in general, random vari-
ation in tastes is sufficient to assure that, in stationary equilibria, each generation’s
savings and asset holding are not restricted to households with the highest dis-
count factor. We thus provide a way of overcoming the difficulty posed by the
analysis of the deterministic model which is both simple and natural.

The apparent shortcomings of representative agent models has increased, in
recent years, the interest in dynamic models with heterogenous agents (e.g., Kir-
man, 1992). For example, in macroeconomic models, the interest in stochastic
heterogenous households economies stems from the persistent inconsistencies be-
tween the solutions attained in complete markets representative agent stochastic
equilibria (based on the Brock-Mirman, 1972, framework) and the observations
from macroeconomic and individual consumption data (Aiyagari, 1994, 1995).
Incorporating precautionary savings and liquidity constraints into a life cycle
model results in a framework which is more compatible with the existing em-
pirical evidence (Deaton, 1992). We see the contribution of the present paper as
a further step in this direction. In particular, our analysis emphasizes the impor-
tance of the variability of the rate of time preference, especially when we consider
future generations (see Laibson, 1997; Schelling, 1995; Weitzman, 1994, 1998)
and its impact on the economy.

Our main idea is rather simple. In steady-state equilibrium the aggregate
capital stock is constant. However, in each generation there are individuals that
inherit capital and whose rate of time preference is sufficiently high that they
dissave. Therefore, to sustain the steady-state equilibrium, it is necessary that
individuals whose rate of time preference is low save and that the resulting capital
accumulation is sufficient to make up the decumulation of the capital stock by
individuals of the first type. If the set of individuals with the lowest rate of time
preference (the most patient households) is small they are unable, by themselves,
to create sufficient additional capital to maintain the steady-state equilibrium. This
implies that the interest rate must reach a level that will induce positive levels
of saving by individuals whose rate of time preference is not the lowest. This
logic implies that, in general, positive saving should take place among individuals
with different rates of time preference. In addition, the stochastic process itself
implies that, in each generation, capital is held by all types of individuals and that
it is possible that individuals that have the same rate of time preference display
different saving behavior. More specifically, given a rate of time preference, it is
possible that individuals who inherit large fortunes dissave while individuals who
inherit small fortunes save. We also show that the differences among discount