Derived factor demand under monopoly

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Received: April 5, 2000; revised version: June 7, 2001

Summary. We generalize the formula provided by Maurice and Ferguson (1973) for derived factor demand in a monopoly by extending it to cross-price effects and taking into account other variables which may, within a general-equilibrium framework, affect demand, such as income. Hopefully, both features increase the applicability of this formula in general-equilibrium analyses.

Keywords and Phrases: Derived factor demand, Monopoly.

JEL Classification Numbers: D21, D42.

1 Introduction

Maurice and Ferguson (1973) provided a neat formula for derived factor demand of a monopoly, cf. Eq. (16) therein. This formula decomposes the price elasticity of a factor of production into a scale and a pure substitution effect and illustrates how these terms are affected by the crucial underlying parameters, i.e., by the elasticities of both the cost and the revenue function. We generalize their formula with respect to two aspects. Firstly, they only consider own-price effects while we also consider cross-price effects. And secondly, we allow for other variables than the output price to have an impact on the demand for the monopoly’s output. By taking into account the impact of other variables, such as income, on derived factor demand, this latter feature hopefully provides the basis for an application of this formula to general-equilibrium analyses.

* I would like to thank Dirk Ebmeyer, Ashley Piggins, and Walter Trockel for helpful comments and suggestions. I am also grateful to an anonymous referee for his/her constructive criticism.
2 The model

Consider a monopolistic firm using a strictly concave, twice continuously differentiable production function

\[ f : \mathbb{R}^n_+ \rightarrow \mathbb{R} : x \mapsto f(x), \]

with positive marginal products

\[ f_i(x) := \frac{\partial f}{\partial x_i}(x) > 0, \quad \forall x \in \mathbb{R}^n_+ \text{ and for all } i = 1, \ldots, n, \]

Let \( w := (w_1, \ldots, w_n) > 0 \) denote the vector of exogenously given factor prices, each of which we assume to be positive, i.e., \( w_i > 0 \) for all \( i = 1, \ldots, n \).

Demand for the firm’s product depends on its price \( p \) and some other variable \( I \), and can be described by some twice continuously differentiable demand function

\[ d : \mathbb{R}^2_+ \rightarrow \mathbb{R}^2_+: (p, I) \mapsto d(p, I) \]

with derivatives

\[ d_p(p, I) := \frac{\partial d}{\partial p}(p, I) < 0, \quad \forall (p, I) \in \mathbb{R}^2_+, \]

and the cross derivative

\[ d_{pp}(p, I) := \frac{\partial^2 d}{\partial p^2}(p, I). \]

For the purpose of the present paper it seems suitable to interpret \( I \) as consumers’ aggregate income. We assume that \( d \) is not ‘too convex’ in the sense that

\[ d_{pp}(p, I) < -2 \frac{d_p(p, I)}{p} \quad \forall (p, I) \in \mathbb{R}^2_+. \]

In the following we shall derive the firm’s profit-maximizing factor demand and investigate its dependence upon changes in factor prices. For this purpose, we find it convenient – although it is not necessary – to decompose the profit-maximization problem of the monopoly into two steps: the cost-minimizing and the final profit maximization problem, the latter of which then solely consists of the choice of the supply level.

3 Cost minimization

Suppose the firm minimizes for any given output level \( Q \) its cost of production. That is, it chooses \( x \in \mathbb{R}^n_+ \) so as to maximize \(-wx\) subject to \( f(x) \geq Q \). Then, the corresponding Lagrangian reads as

\[ \mathcal{L}^e(x_1, \ldots, x_n, \lambda^e, Q) = -wx + \lambda^e(f(x) - Q), \]

with \( \lambda^e \geq 0 \).

The quasi-saddle-point condition for this problem can be written as

\[ -w_i + \lambda^e f_i(\hat{x}) \leq 0, \quad \left[ -w_i + \lambda^e f_i(\hat{x}) \right] \hat{x}_i = 0, \quad \forall i = 1, \ldots, n \]

\[ f(\hat{x}) - Q \geq 0, \quad \left[ f(\hat{x}) - Q \right] \hat{\lambda}^e = 0, \quad \hat{x} \geq 0, \quad \hat{\lambda}^e \geq 0. \]

The analysis can easily be extended to any arbitrary number of variables affecting demand.