Robust controller design with hard constraints on the control signal

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Contents The paper deals with robust controller design for motion control systems in case of constraints on the control signal. Furthermore, in order to enhance the dynamic performances of the designed control system – in case of limitation of the control signal – gain-scheduled control strategy is considered. The proposed robust controller design approach is verified by experimental results in case of a vector controlled brushless d.c. drive.

1 Introduction
Controller design for systems with hard constraints is a quite vivid area of research, due to the fact that most practical control problems are dominated by constraints on the control signal [1, 2]. Exceeding the prescribed bounds causes unexpected behavior of the system – large overshoot, low performance or (in the worst case) instability.

Furthermore, process models are always inaccurate – even extremely detailed models may contain unknown or slowly changing physical parameters – so the controller has to manage the difference between the model (used for design) and the real plant. Bridging the gap between model and real plant is the field of robust controller design.

A general framework for the design of control systems subject to hard constraints is presented in [3]. The design procedure is based on the $H_\infty$ loop shaping and relies on the calculation of the maximum possible control amplitude for a class of reference signals, bounded in amplitude and rate.

Controller designs that take into account the saturation effect a-priori are usually separated into two categories:

- designs that prevent saturation of the control signal and therefore enjoy a linear framework (as long as plant and controller are linear);
- methods that allow saturation and therefore facing a nonlinear setup.

The paper focuses on the first category – saturation avoiding – philosophy. To solve the constraint control problem, implicitly has to restrict the amplitude as well as the rate of the external signals. In many practical situations, this is a more accurate description, than without rate restriction. For example, in case of tank, not only the liquid-level is bounded (by the tank’s height) in addition the liquid-level cannot change arbitrarily fast. During our investigation we will consider a single-input single-output (SISO) system, although the procedure can be extended for multi-input multi-output (MIMO) systems [4, 5].

2 Theoretical backgrounds
Definition 1 (Admissible reference signal) Let $\bar{R}, \hat{R} \in \mathbb{R}$ with $R \geq 0$ and $\bar{R} \geq 0$. A continuous piecewise differentiable reference signal $r(t)$, fulfilling $r(t) = 0$ for all $t \leq 0$ is called $(\bar{R}, \hat{R})$ admissible, when the following properties hold:

$$|r(t)| \leq \bar{R} \quad \forall \ t > 0$$

$$|\dot{r}(t)| \leq \hat{R} \quad \forall \ t > 0$$

The set of all $(\bar{R}, \hat{R})$, admissible reference signals, is denoted by $\mathcal{A}(\bar{R}, \hat{R})$.

Definition 2 (Maximum control amplitude) Given the internally stable standard control loop as in Fig. 1. The maximum control amplitude is the $H_\infty$-norm of the control signal:

$$u_{\text{max}} = \|u\|_{\infty} = \sup_{t} |u(t)|, \quad r \in \mathcal{A}(\bar{R}, \hat{R})$$

and let us note with $G_p(s)$ the transfer function of the plant and $K(s)$ the controller’s transfer function.

The transfer functions $S(s)$ and $T(s)$ are known as the sensitivity function and the complementary sensitivity function, and are defined as follows:

$$S(s) = (1 + K(s)G_p(s))^{-1} \quad \text{and} \quad T(s) = 1 - S(s)$$

The $W_S(s)$ and $W_T(s)$ are the related weighting functions. Moreover, let us denote the transfer function from reference $r$ to control signal $u$ with:

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The augmented plant

\[ H(s) = \frac{K(s)}{1 + K(s)G_p(s)} \]  

(4)

**Theorem 1 (Glover 1987)** Let \( H(s) \) be asymptotically stable and strictly proper transfer function with McMillan degree \( n_i \), input \( r \), output \( u \). Then the following inequality holds:

\[ \|u\|_\infty \leq 2n\|H\|_\infty \|r\|_\infty \]  

(5)

upper bound for maximum control amplitude [6].

As a remark, the previous theorem can be extended to the case of proper transfer functions too [4].

According with the previous theorem an explicit relation exists between the \( H_\infty \)-norm of the transfer function \( H(s) \) (i.e. the maximum over all frequencies of the largest singular value), and the maximum control amplitude.

The next question is, if there exists a relation between the singular values of the weights and those of the transfer function \( H(s) \)? This is stated in [7], that decreasing the weights decreases the transfer function \( H(s) \) (in terms of the largest singular value). Hence is observed that decreasing the \( H_\infty \)-norm of the weights decreases an upper bound for the maximum control amplitude.

Within the loop shaping, this has the following consequences:

- we have to decrease the maximum singular value of the design weights in the frequency range where the \( H_\infty \)-norm of \( H(s) \) transfer function is large;
- in case of too low maximum control signal, we have to increase the maximum singular value in that frequency range.

A loop shaping algorithm for controller design in case of systems with constraints is described in [3, 4].

**Algorithm 1 (Loop shaping design)** Given a plant \( G_p(s) \), with restrictions on the reference signal and the desired control amplitude \( u_{max} \), the loop shaping might be performed in the following way:

1. choose the design weights \( W_s(s) \) and \( W_r(s) \);
2. design the controller for the shaped plant and compute the stability margin;
3. decide weather the design-objectives are fulfilled or not:

- is the stability margin large enough;
- are the performance-objectives fulfilled;
- does \( u(t) \leq u_{max} \) hold for all admissible reference signals \( (R, \dot{R}) \).

4. If not, adapt weight \( W_s(s) \) and go to step 1.

The previous algorithm can be applied for motion control systems too, in order to design a robust controller that prevent the saturation of the control signal.

**3 Identification of the investigated brushless d.c. drive**

A simplified block diagram of the investigated vector controlled brushless d.c. (BLDC) drive is presented in Fig. 2 and the nominal parameters of the motor are given in Table 1.

The brushless d.c. drive is identified using the well-known auto-regressive exogenous (ARX) method [8, 9].

The input signal is the \( i_R^d \) reference current and the output is the motor speed. As input signal the pseudo-random binary signal (PSBS) is applied due to it’s wide frequency range characteristics. The input and the output signals are shown in Fig. 3.

Defining a frequency range for identification as \( \omega_{id} \) the sampling time \( T_s \) is selected according to Eq. (6):

\[ \frac{2\pi}{1007T_s} < \omega_{id} < \frac{2\pi}{5T_s} \]  

(6)

In our case \( T_s = 0.001 \) s, that means

\[ 50 \text{ rad/s} < \omega_{id} < 1250 \text{ rad/s} \]

The identified mathematical model is a 15th order model, therefore model order reduction is performed based on balanced truncation technique.

Based on the balanced truncation technique, a 2nd order reduced model is obtained. A comparison between the identified mathematical model and the reduced order model is presented in Fig. 4 (gain plot), where the dotted line corresponds to the high order model (15th order model) and the solid line to the reduced order model (2nd order model).

The transfer function of the reduced order model can be written as:

\[ G_n(s) = \frac{K_{gain}}{(sT_e + 1)(sT_m + 1)} \]  

(7)

where: \( K_{gain} = 100 \text{ rad/s/A} \), \( T_e = 1/800 \) s and \( T_m = 1/2.5 \) s.

In the controller design process different model parameter uncertainties can be considered [10]. All models used in feedback design should include some unstructured uncertainty to cover modeled uncertainty, particularly at high frequency. Let us focus on the multiplicative uncertainty, derived from the following Eq. (8):

\[ \Delta_m(s) = \frac{G_p(s) - G_n(s)}{G_n(s)} \]  

(8)

where \( G_n(s) \) denotes the nominal plant. The gain plot related to the deduced multiplicative uncertainty is shown in Fig. 5, solid line. The deduced mathematical model is accurate enough in the 50–1250 rad/s frequency range. According to a robust control perspective, in order to take