Blowing up: application to $G^2$-continuous 8-sided filling patch

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Summary. We propose a method for filling a $n$-sided hole, $5 \leq n \leq 8$, that interpolates $n$ connected boundary curves of a given net of patches. This method allows the joining with patches defined in many different ways. A new class of blowing up pole-functions is introduced in order to build a $G^2$-continuous $n$-sided filling surface. This filling surface is in one piece, image of $[0,1]^2$.

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1 Introduction. General framework. Blowing up pole-functions

In CAD-CAM the industrial designers define the main parts of the pieces to manufacture, complying with functional and aesthetic criteria. These parts are then linked together by joining patches which play a pivotal role within compatible manufacturing constraints. We shall here focus on the joining problem referred to as a filling patch. This means filling a $n$-sided hole ($5 \leq n \leq 8$) in order to connect a net of given adjacent patches with geometric continuity. To do so, we build a $n$-sided patch in one piece defined on $[0,1]^2$ in order to join it $G^2$ (or more)-continuously with the $n$ given patches along boundary curves. The patches or surfaces defining these boundary curves can be of different kinds with their suitable representations: Bézier-de Casteljau polynomial patches, (SBR) rational patches, or more general...
parametric surfaces or implicit surfaces. The filling surface is always defined on a rectangular domain and not on a \( n \)-sided (\( n \leq 8 \)) polygonal domain as \([18],[21],[25]\) did. In \([10],[14]\) we introduced pole-functions in order to create a \( G^3 \)-continuous rectangular filling patch (\( n = 4 \)). However this method cannot be applied to fill a \( n \)-sided hole with \( n \geq 5 \) with a filling surface defined on \([0,1]^2\). In order to tackle this more complex problem we propose to use a new class of pole-functions called blowing up pole-functions. This approach briefly initiated in short notes \([13],[12]\) is further developed here. It allows us to create such a \( n \)-sided (\( n \leq 8 \)) \( G^1 \) or \( G^2 \) (or more)-continuous filling surface. Moreover, contrary to \([18],[20],[25]\) that new filling surface can be built even if the partial derivatives of order one or two at the corners are not compatible. For instance, we assume that the first partial derivatives of the given surfaces at the corners are not systematically on a same plane.

Section 1 defines pole-function blowing up at a vertex of \([0,1]^2\).

In Sect. 2, we shall be more precise about pole-function blowing up at vertex \((0,0)\) in order to define a 5-sided \( G^1 \)-continuous filling surface. The fifth boundary curve will thus be the image of the vertex \((0,0)\) of \([0,1]^2\). This is an arbitrary choice. We could choose one of the other three vertices (i.e. \((1,0)\), \((1,1)\), \((0,1)\)). We give sufficient conditions (Proposition 1) that the pole-functions and the pole-functions blowing up at \((0,0)\) have to satisfy in order to obtain the \( G^1 \)-continuity. We propose a model (Proposition 2) which allows us to create these 5-sided \( G^1 \)-continuous filling surfaces. Fig. 6 illustrates the above mentioned purpose: we can build a 5-sided \( G^1 \)-continuous filling surface in spite of the fact that no common tangent plane appears at the two corners.

By modifying the pole-functions blowing up at \((0,0)\) we shall be able in Sect. 3 to define a 5-sided \( G^2 \)-continuous filling surface.

Section 4 applies this method and gives a model to create an 8-sided \( G^2 \)-continuous filling surface. In this case, the three additional boundary curves are the images of the three remaining vertices (i.e. \((1,0)\), \((1,1)\), \((0,1)\)) of \([0,1]^2\).

For \( i = 0, \ldots, n \), let \( B_i^n (t) = \binom{n}{i} (1 - t)^{n-i} t^i \) be the Bernstein’s polynomials, then we have the following definition of pole-functions.

**Definition 1** Let \( n \) and \( p \) be two integers and \( S \) be a surface parametrized by

\[
f : D \subset \mathbb{R}^2 \to \mathbb{R}^3
\]

\[
(u,v) \mapsto f(u,v) = \sum_{i=0}^{n} \sum_{j=0}^{p} B_i^n (u) B_j^p (v) f_{ij} (u,v),
\]

where \( D \) is any subset of \( \mathbb{R}^2 \). Then we called functions \( f_{ij} : D \subset \mathbb{R}^2 \to \mathbb{R}^3 \) pole-functions.