The Topological Open String Wavefunction

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Abstract: We show that, in local Calabi–Yau manifolds, the topological open string partition function transforms as a wavefunction under modular transformations. Our derivation is based on the topological recursion for matrix models, and it generalizes in a natural way the known result for the closed topological string sector. As an application, we derive results for vacuum expectation values of $1/2$ BPS Wilson loops in ABJM theory at all genera in a strong coupling expansion, for various representations.

1. Introduction

Topological string theory on Calabi–Yau (CY) manifolds has been an important source of results in string theory, gauge theory and mathematics (see for example [1–5] for reviews). In the so-called local case, where the CY is non-compact, the theory can be solved exactly, by using for example large $N$ techniques in matrix models [6–8] or the theory of the topological vertex [9].

Closed topological string amplitudes satisfy many interesting properties. In the local case, and from the B-model point of view, they can be regarded as holomorphic objects associated to an algebraic curve or Riemann surface. They depend on a choice of “symplectic frame”, i.e., on a choice of symplectic basis for the homology of the Riemann surface, and they turn out to have non-trivial transformation properties under a change of basis or modular transformation. Equivalently, one can introduce a non-holomorphic dependence in the amplitudes, which is governed by the holomorphic anomaly equations of [10]. As shown in [11], the transformation properties of the closed string amplitudes can be derived from the fact that the total closed string partition function (summed over all genera) is a wavefunction [12]. Modular transformations correspond to canonical transformations, which lift quantum-mechanically to integral transforms of the wavefunction. Therefore, a change of symplectic basis leads to an integral transform of the topological closed string partition function.
These properties of the closed topological string amplitudes can be also derived by using the solution of the B-model in terms of matrix integrals [6–8]. This solution is based on the topological recursion of Eynard and Orantin [13], which encodes as well the modular properties of the resulting amplitudes. It was shown in [14] that the modular behavior of the closed string amplitudes, as deduced from the topological recursion, agrees indeed with the wavefunction behavior of the partition function found in [11,12].

The topological recursion of [13] also gives a method to compute the modular transformation of open string amplitudes. In this paper, we show that these properties can be summarized by saying that the total open string partition function transforms as a wavefunction. This generalizes the results of [11,12] to the open sector, since the closed string partition function can be regarded as a specialization of the open string partition function where all the open moduli are set to zero.

The wavefunction behavior of the open string partition function has practical applications, since it makes it possible to relate in a precise way open string amplitudes in different frames. One interesting situation where this can be used is the calculation of vacuum expectation values (vevs) of 1/2 BPS Wilson loops [15] in ABJM theory [16]. These vevs can be computed by localization, in terms of a matrix model [15,17]. It turns out that they are given by open topological string amplitudes in a non-compact CY, local \( \mathbb{P}^1 \times \mathbb{P}^1 \) [18], but in the so-called orbifold frame [19]. As an application of the main result of this paper, we obtain results for the vevs of 1/2 BPS Wilson loops by first performing the calculation in the large radius frame, and then using the fact that the open string partition function is a wavefunction. We find in this way all-genus results for vevs of 1/2 BPS Wilson loops as integral transforms of topological string amplitudes at large radius. These expressions are exact in \( k \), the coupling of ABJM theory, but they are expanded around the strong coupling limit. They correspond to the M-theory expansion of the amplitudes discussed, in for example, [20,21]. In particular, we rederive in this way the result for 1/2 BPS Wilson loop vev with winding \( n \) derived in [21] in the M-theory regime, and we extend it to other representations. Our method also makes it possible to calculate systematically worldsheet instanton corrections, which are difficult to obtain in the Fermi gas approach of [21].

The wavefunction behavior of the open string amplitudes has been addressed before. In [22–24], the behavior of the open string partition function has been studied as one changes the open moduli, although as far as we know there is no general statement for this behavior. The paper [25] studies the wavefunction behavior of the open string partition function in the compact CY case.

This paper is organized as follows. In Sect. 2 we review the definition and construction of topological open string amplitudes and the topological recursion of Eynard and Orantin. In Sect. 3 we derive our main result, namely, we show that the total, topological open string partition function transforms as a wavefunction under modular transformations. In Sect. 4 we use our main result to obtain expressions for 1/2 BPS Wilson loop vevs at all orders in the genus expansion and expanded at strong coupling. Finally, in Sect. 5 we end up with some conclusions and prospects for future work.

2. Open Topological String Amplitudes and Topological Recursion

2.1. Open topological string amplitudes. In this paper we will study open topological string amplitudes in local CY geometries. There are two types of local CYs which are particularly interesting. The first ones are of the form

\[
uv = H(x, y), \quad H(x, y) = y^2 - (W'(x))^2 + f(x),
\]  

(2.1)