Vacuum Radiation and Symmetry Breaking in
Conformally Invariant Quantum Field Theory *

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Abstract: The underlying reasons for the difficulty of unitarily implementing the whole conformal group $\text{SO}(4, 2)$ in a massless Quantum Field Theory (QFT) on Minkowski space are investigated in this paper. Firstly, we demonstrate that the singular action of the subgroup of special conformal transformations (SCT), on the standard Minkowski space $M$, cannot be primarily associated with the vacuum radiation problems, the reason being more profound and related to the dynamical breakdown of part of the conformal symmetry (the SCT subgroup, to be more precise) when representations of null mass are selected inside the representations of the whole conformal group. Then we show how the vacuum of the massless QFT radiates under the action of SCT (usually interpreted as transitions to a uniformly accelerated frame) and we calculate exactly the spectrum of the outgoing particles, which proves to be a generalization of the Planckian one, this recovered as a given limit.

1. Introduction

The conformal group $\text{SO}(4, 2)$ has ever been recognized as a symmetry of the Maxwell equations for classical electro-dynamics [C-B], and more recently considered as an invariance of general, non-abelian, massless gauge field theories at the classical level. However, the quantum theory raises, in general, serious problems in the implementation of conformal symmetry, and much work has been devoted to the study of the physical reasons for that (see e.g. [Fr]). Basically, the main trouble associated with this quantum symmetry (at the second quantization level) lies in the difficulty of finding a vacuum, which is stable under special conformal transformations acting on the Minkowski space in the form:

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\[ x^\mu \to x'^\mu = \frac{x^\mu + e^\mu x^2}{\sigma(x, c)}, \quad \sigma(x, c) = 1 + 2cx + c^2x^2. \]  \hspace{1cm} (1)

These transformations, which can be interpreted as transitions to systems of relativistic, uniformly accelerated observers (see e.g. [H]), cause vacuum radiation, a phenomenon analogous to the Fulling-Unruh effect [Fu, U] in a non-inertial reference frame. To be more precise, if \( a(k) \), \( a^*(k) \) are the Fourier components of a scalar massless field \( \phi(x) \), satisfying the equation

\[ a^\mu \partial_\mu \phi(x) = 0, \] \hspace{1cm} (2)

then, the Fourier components \( a'(k) \), \( a'^*(k) \) of the transformed field \( \phi'(x') = \sigma^{-1}(x, c)\phi(x) \) by (1) (\( l \) being the conformal dimension) are expressed in terms of both \( a(k) \), \( a^*(k) \) through a Bogolyubov transformation

\[ a'(\lambda) = \int dk \left[ A_\lambda(k)a(k) + B_\lambda(k)a^*(k) \right]. \] \hspace{1cm} (3)

In the second quantized theory, the vacuum states defined by the conditions \( \hat{a}(k)|0\rangle = 0 \) and \( \hat{a}'(0')|0\rangle = 0 \), are not identical if the coefficients \( B_\lambda(k) \) in (3) are not zero. In this case the new vacuum has a non-trivial content of untransformed particle states.

This situation is always present when quantizing field theories in curved space as well as in flat space, whenever some kind of global mutilation of the space is involved. This is the case of the natural quantization in Rindler coordinates [BD], which leads to a quantization inequivalent to the normal Minkowski quantization, or that of a quantum field in a box, where a dilatation produces a rearrangement of the vacuum [Fu]. Nevertheless, it must be stressed that the situation for SCT is more peculiar. The rearrangement of the vacuum in a massless QFT due to SCT, even though they are a symmetry of the classical system, behaves as if the conformal group were spontaneously broken, and this fact can be interpreted as a kind of topological anomaly.

Thinking of the underlying reasons for this anomaly, we are tempted to make the singular action of the transformations (1) in Minkowski space responsible for it, as has been in fact pointed out in [GU]. However, a deeper analysis of the interconnection between symmetry and quantization will reveal a more profound obstruction to the possibility of implementing unitarily SCT in a generalized Minkowski space, free from singularities, when conformally invariant fields are forced to evolve in time. This way, the quantum time evolution itself destroys the conformal symmetry, leading to some sort of dynamical symmetry breaking which preserves the Weyl subgroup (Poincaré + dilatations).

This obstruction is traced back to the impossibility of representing the entire \( SO(4, 2) \) group unitarily and irreducibly on a space of functions depending arbitrarily on \( \vec{z} \) (see e.g. [Fr]), so that a Cauchy surface determines the evolution in time. Natural representations, however, can be constructed by means of wave functions having support on the whole space-time and evolving in some kind of proper time.

From the point of view of particle quantum mechanics (or "first" quantization), the free arguments of wave functions in the configuration-space "representation" correspond to half of the canonically conjugated variables in phase space (or the classical solution manifold), and this phase space is usually defined as a co-adjoint orbit of the basic symmetry group characterizing the physical system. Thus, for instance, for the Galilei or Poincaré group the phase space associated with massive spinless particles has dimension 6 and the corresponding wave functions in configuration space have the time variable...