Thermal radiation effect on non-Darcy natural convection with lateral mass transfer

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Abstract A boundary layer analysis has been presented to study the influence of thermal radiation and lateral mass flux on non-Darcy natural convection over a vertical flat plate in a fluid saturated porous medium. Forchheimer extension is considered in the flow equations, and the Rosseland approximation is used to describe the radiative heat flux in the energy equation. Similarity solution for the transformed governing equations is obtained and the combined effect of thermal radiation and fluid suction/injection on the heat transfer rate is discussed. Numerical results for the details of the velocity and temperature profiles as well as Nusselt number have been presented.

List of symbols
A constant
C empirical constant
CT temperature difference
d pore diameter
f non-dimensional stream function
fw non-dimensional mass flux parameter
g gravitational constant
K permeability of the porous medium
k3 dispersion thermal conductivity
ke effective thermal conductivity
Nu local Nusselt number
p pressure
q local heat flux
q0 radiative heat flux
R radiation parameter
Rax Rayleigh number
T temperature
u, v velocity components in the x and y directions
x, y cartesian coordinates
\( \rho \) fluid density
\( \mu \) viscosity
v fluid kinematic viscosity
\( \alpha \) molecular thermal diffusivity
\( \beta \) thermal expansion coefficient
\( \delta_T \) boundary layer thickness evaluated at \( \theta(\eta) = 0.001 \)
\( \eta \) similarity parameter
\( \psi \) dimensional stream function
\( \theta \) non-dimensional temperature
\( \sigma \) Stefan–Boltzmann constant
\( \chi \) the mean absorption coefficient

Subscripts
w evaluated on the wall
\( \infty \) evaluated at the outer edge of the boundary layer

1 Introduction
Study of convection boundary layer flow in porous media has received considerable interest, because of its wide applicability in energy, such as geothermal energy technology, petroleum recovery, filtration processes, packed bed reactors and underground disposal of chemical and nuclear waste. Most of the works dealing with convective heat transfer in porous media have been motivated by geothermal applications. Cheng [1] presented a comprehensive review about heat transfer in geothermal systems. The effects of fluid suction and injection on the natural convection heat transfer in a Darcian fluid saturated porous medium has been studied by the same author [2] and Merkin [3]. In the references [4–6] Forchheimer extension are used to study the non-Darcy natural convection from the vertical wall. Effects of thermal dispersion and lateral mass flux on non-Darcy natural convection over a vertical flat plate in a fluid saturated porous medium was studied by Murthy and Singh [7].


The present investigation is devoted to study the effects of radiation and lateral mass flux on Forchheimer natural convection over a vertical flat plate in a fluid saturated porous medium. The suction/injection velocity distribution has been assumed to have power function from \( Ax^l \),

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where \( x \) is the distance from the leading edge and the wall temperature distribution is assumed to be uniform. Similarity solution is possible when \( l = -1/2 \). The Rosseland approximation is used to describe the radiative heat flux in the energy equation.

2

Analysis

Let us consider the non-Darcy natural convection-radiation flow and heat transfer over a semi infinite vertical surface in a fluid saturated porous medium. Figure 1 shows the flow model and coordinate system. The axial and normal coordinates are \( x \) and \( y \) with the origin at the leading edge of the plate. The radiative heat flux in the \( x \)-direction is considered negligible in comparison with that in the \( y \)-direction (Sparrow and Cess [14]). The isothermal hot wall is assumed to be permeable with lateral mass flux in the form \( \nu_w = Ax^2 \). The normal component of the velocity near the boundary is small as compared with the other component of the velocity and the derivatives of any quantity in the normal direction are large compared with derivatives of the quantity in direction of the wall. Under these assumptions, the governing equations for this problem are:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  

(1)

\[
u + \frac{C \sqrt{K}}{v} u |u| = -\frac{K}{\mu} \left( \frac{\partial p}{\partial x} + \rho g \right)
\]  

(2)

\[
\frac{\partial p}{\partial y} = 0
\]  

(3)

\[
u \frac{\partial T}{\partial x} + \nu \frac{\partial T}{\partial y} = \alpha \left( \frac{\partial^2 T}{\partial y^2} - \frac{1}{k_e} \frac{\partial q^r}{\partial y} \right)
\]  

(4)

along with the boundary conditions

\[
y = 0: \nu_w(x) = Ax^2, \quad T_w = \text{const.}
\]  

\[
y \to \infty: u = 0, \quad T \to T_\infty
\]  

(5)

where \( u \) and \( v \) are the velocity components in the \( x \) and \( y \)-directions, respectively, \( \alpha \) is the equivalent thermal diffusivity, \( k_e \) is the effective thermal conductivity of the saturated porous medium, \( \rho \) is the pressure, \( T \) is the temperature, \( K \) is the permeability constant, \( C \) is an empirical constant, \( \beta \) is the thermal expansion coefficient, \( \mu \) is the viscosity of the fluid, \( \rho \) is the density, and \( g \) is the acceleration due to gravity. The quantity \( q^r \) on the right hand side of Eq. (4) represents the radiative heat flux in the \( y \)-direction. The radiative heat flux term is simplified by the Rosseland approximation (cf. Sparrow and Cess [14]) and is as follows

\[
q^r = -\frac{4\sigma \partial T^4}{3\chi \partial y}
\]  

(7)

where \( \sigma \) and \( \chi \) are the Stefan–Boltzmann constant and the mean absorption coefficient.

By substituting Eq. (5) into Eqs. (2) and (3), eliminating the pressure and the velocity components \( u \) and \( v \) can be written in terms of stream function \( \psi \) as:

\[
u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x}, \quad \text{one obtains:}
\]  

\[
\frac{\partial^2 \psi}{\partial y^2} + \frac{C \sqrt{K}}{v} \frac{\partial \psi}{\partial y} \left( \frac{\partial \psi}{\partial y} \right)^2 = \frac{Kg\beta}{\mu} \frac{\partial T}{\partial y}
\]  

(8)

\[
\frac{\partial \psi}{\partial y} \frac{\partial T}{\partial y} - \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial x} = \alpha \left( \frac{\partial^2 T}{\partial y^2} - \frac{1}{k_e} \frac{\partial q^r}{\partial y} \right)
\]  

(9)

Introducing the similarity variable and similarity profiles

\[
\eta = \frac{\psi}{\sqrt{\alpha Ra_x}} \quad \text{and} \quad f(\eta) = \frac{\psi}{\sqrt{\alpha Ra_x}}, \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}
\]  

(10)

where \( Ra_x \) is the modified Rayleigh number,

\[
Ra_x = \frac{(Kg\beta(T_w - T_\infty))}{(\chi \nu d)^{1/2}}
\]

The problem statement then becomes:

\[
f'''' + 2F_0 f'' f''' - \theta' = 0
\]  

(11)

\[
\theta'' + \frac{1}{2} f' + \frac{1}{2} R[(C_T + \theta')^3 \theta'] = 0
\]  

(12)

where the parameter \( F_0 = (C \sqrt{K} \chi)/\nu d \) represents the structural and thermophysical properties of the porous medium, the radiation parameter is defined by \( R = (4\sigma(T_w - T_\infty)^3)/(\chi \cdot k_e) \), \( C_T = T_\infty/(T_w - T_\infty) \) is the temperature difference, and \( Ra_d = (Kg\beta(T_w - T_\infty) d)/(\chi \nu d) \) is the pore diameter dependent Rayleigh number which describes the relative intensity of the buoyancy force, such that \( d \) is the pore diameter. The boundary conditions become

\[
f(0) = f_w, \quad \theta(0) = 1, \quad f'(\infty) = \theta'(\infty) = 0
\]  

(13)

From the definition of the stream function, the velocity components become

\[
u = \frac{x}{2} \frac{\partial f'}{\partial x}
\]  

(14)

\[
u = -\frac{x}{2} \frac{\partial f'}{\partial x}
\]  

(15)

On the wall (\( \eta = 0 \)) Eq. (15) becomes

\[
u_w(x) = -\frac{x}{2} \frac{\partial f'}{\partial x}
\]  

(16)