Experimental investigation of heat conduction in wet sand

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Abstract There has been a lasting discussion in the literature on the validity of Fourier’s heat conduction law. The main concern is the inherent infinite propagation velocity of temperature changes. Some authors hold that due to this problem the law should fail for certain materials or on extreme time scales. Then, to reconcile experiment with theory Fourier’s equation would need an extension to a hyperbolic form. To contribute to this matter we report on unsteady heat conduction experiments in wet sand for which hyperbolic behaviour (Kaminski 1990) was documented. Our results clearly salvage Fourier’s law without giving any explanation on why a theory with a serious inconsistency can work so well.

List of symbols

- \( a \) thermal diffusivity [m\(^2\)/s]
- \( A_t \) surface tube [m\(^2\)]
- \( c_t \) specific heat of heating tube [J/kg\(\cdot\)K]
- \( h \) length of heating tube [m]
- \( K \) Fourier number, determined by the finite differences (time/space)
- \( m \) number of increments, time
- \( m_t \) mass of heating tube [kg]
- \( n \) number of increments, space
- \( Q \) heat flux [W]
- \( r \) radius [m]
- \( s \) thickness, \( r_2 - r_1 \) [m]
- \( T \) temperature [\(^\circ\)C]
- \( t \) time [s]
- \( T_\infty \) exterior temperature [\(^\circ\)C]
- \( T_0 \) initial temperature [\(^\circ\)C]
- \( \alpha \) convective heat-transfer coefficient [W/m\(^2\)\(\cdot\)K]
- \( \Delta r \) increment in space [m]
- \( \Delta t \) increment in time [s]
- \( \lambda \) thermal conductivity [W/m\(\cdot\)K]
- \( \mu \) fraction of moisture
- \( \rho \) density [kg/m\(^3\)]
- \( \tau \) relaxation time [s]

1 Introduction

Unsteady heat conduction processes in homogeneous solid bodies or stagnant fluids are treated by Fourier’s differential equation

\[
\frac{\partial T}{\partial t} = a \Delta T .
\] (1)

Any solution, obtained for given boundary and initial conditions, predicts which temperature \( T \) is to be expected in space and time. Accordingly, \( a \) is called the thermal diffusivity with the dimension m\(^2\)/s. A peculiarity of this equation is that it has been extremely successful although it contains a serious inconsistency concerning the propagation speed of temperature changes. Looking at the solutions it becomes clear that an infinite propagation speed is inherently involved. From a physical point of view, regarding molecular interaction or electron motion as transport mechanisms this is impossible.

In order to resolve this paradox attempts were made to modify the equation such that Fourier’s equation appears as a limiting case of a more general formulation. Cattaneo (1958) and Vernotte (1958) suggested an additional term such that

\[
\tau \frac{\partial^2 T}{\partial t^2} + \frac{\partial T}{\partial t} = a \Delta T.
\] (2)

One may as well understand this equation as the wave equation with \( \partial^2 T/\partial t^2 \) as an additional term. This helps to see how a finite wave speed comes in (\( a/\tau \) would be the squared wave speed in the wave equation). In scaling the new term the so called relaxation time \( \tau \) determines the strength of the wave influence. For \( \tau \rightarrow 0 \) Eq. (1) is retrieved. Mathematically Eq. (1) is parabolic while Eq. (2) is hyperbolic.

The hyperbolic heat conduction equation has seen a number of successful solutions for various physical situations (e.g. Wiggert 1977; Kar et al. 1992; Lewandowska et al. 1998). However, there is also some doubt emerging on the validity of the equation. For example, Taitel (1972) treats the case of an initially isothermal plate heated evenly from both sides. He finds that the temperature in the centre of the plate is elevated above the side temperature. Barletta and Zanchini (1997a) come to an analogous result.
for a cylinder. Whether this finding violates the second law of thermodynamics is extensively discussed by Barletta and Zanchini (1997b).

Experimental work referring to the hyperbolic character is scarce. The most frequently quoted work is that of Kaminski (1990) and Mitra et al. (1995). It is much in favour of the hyperbolic equation and has given rise to more theoretical work (Antaki 1997). Mitra et al. (1995) look at heat conduction in processed meat where they find relaxation times of up to 17 s. Kaminski (1990) investigates porous media such as wet sand. For a wetness fraction of 1% in sand he determines a relaxation time of 20 s.

Relaxation times of that magnitude certainly have an unsettling effect on the confidence in Fourier’s heat conduction. Having used Fourier’s law in various applications without any indication of failure, we decided to carry out some experiments ourselves with wet sand at wetness fractions from 0% to 8.5% by weight. Along with the experiments we measured the material properties necessary for calculation.

We use a cylindrically symmetric system (Fig. 1) with an electrical heat source along the axis. We begin with the formulation of the numerical model and proceed with the description of the experiments thereafter.

2 Model calculations

For a cylinder of homogenous material and infinite length Eq. (1) reads

$$\frac{\partial T}{\partial t} = a \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right)$$  \hspace{1cm} (3)

In accordance with our experiments the equation has to be solved under the following initial and boundary conditions. The inner boundary is the surface of a thin tube of radius $r_1$ which is resistance heated by applying a voltage to an embedded wire. Before the heating is switched on everything is at the initial temperature $T_0$. The heating starts at $t = 0$. Part of the heat remains in the tube itself while the main part is transferred across the tube’s surface into the heat conducting material which is sand in our case. Due to a comparatively large thermal diffusivity of the tube (metal) the tube’s temperature $T_1$ is assumed to be uniform.

The outer boundary at $r_2$ consists of a thin walled PVC tube holding the sand. Neglecting its heat capacity the heat flux entering the tube leaves it at the outer surface at the same rate. We may write now

$$T - T_0 = T_\infty - T_0 + \frac{\dot{Q} r_1}{A \lambda} \left( \frac{\lambda}{\lambda - r_2} + \ln \frac{r_2}{r} \right)$$

$$- 2 \sum_{n=1}^\infty \left( \frac{\mu_n \cdot r_2}{\lambda - r_2} (T_\infty - T_0) \right) \cdot \left( J_0 \left( \frac{\nu_n}{T} \right) N_1 \left( \frac{\nu_n}{T} \right) - J_1 \left( \frac{\nu_n}{T} \right) N_0 \left( \frac{\nu_n}{T} \right) \right)$$

$$\cdot e^{-\frac{\mu_n r_2}{T}}$$

(5)

The quantities $E$ and $G$ stand for

$$E = J_1 \left( \frac{\mu_n \cdot r_2}{s} \right) \cdot N_1 \left( \frac{\mu_n \cdot r_1}{s} \right)$$

$$- J_1 \left( \frac{\mu_n \cdot r_1}{s} \right) \cdot N_1 \left( \frac{\mu_n \cdot r_2}{s} \right)$$

$$G = J_0 \left( \frac{\mu_n \cdot r_2}{s} \right) \cdot N_1 \left( \frac{\mu_n \cdot r_1}{s} \right)$$

$$- J_1 \left( \frac{\mu_n \cdot r_1}{s} \right) \cdot N_0 \left( \frac{\mu_n \cdot r_2}{s} \right).$$

(6)

The eigenvalues $\mu_n$ have to be numerically evaluated in principle from $n = 1$ to $\infty$ from