Performance analysis of extended surfaces subjected to fouling

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Abstract The time dependent performance of extended surfaces subjected to fouling is addressed in this work. Where fins are used for augmenting boiling heat transfer, the interaction of local values of temperature excess, fouling resistance and surface characteristics of the deposit can be quite complex. Taking typical asymptotic fouling growth parameters from literature for reverse solubility salts, three kinds of fin geometry are analysed – rectangular, triangular and annular. For various values of the fin parameter mL, the temperature distribution and variation of fouling resistance are obtained as a function of time. To interpret the performance of a fouled fin, a new term ‘cleanliness efficiency’ is introduced. The necessity of choosing an optimal value of mL for the fin is also highlighted here. It is shown that for all three fin configurations, cleanliness efficiency differs little, thus simplifying the geometry dependence. The approach set out in this work will help in the design of finned heat exchangers subjected to fouling and thereby minimise their overdesign.

List of symbols

- $A$: area, m$^2$
- $A_o$: area of the fin at the base, m$^2$
- $E$: activation energy, KJ/kg·mol
- $h$: convective heat transfer co-efficient, W/m$^2$·K
- $h_a$: apparent heat transfer co-efficient, W/m$^2$·K
- $L$: length of the fin, m
- $k$: thermal conductivity of fin material, W/m·K
- $P_0$: perimeter of fin at base, m
- $r_i$: inner radius of annular fin, m
- $r_o$: outer radius of annular fin, m
- $R$: universal gas constant, KJ/kg·mol·K
- $R_f$: fouling resistance, m$^2$·K/W
- $R_f^s$: asymptotic fouling resistance, m$^2$·K/W
- $S$: surface area, m$^2$
- $t$: time, s
- $T$: fin temperature, K
- $T_b$: base temperature, K
- $T_\infty$: fluid temperature, K
- $T_s$: surface temperature, K
- $T_{s_n}$: surface temperature at nth instant, K
- $x$: distance from the base, m
- $X$: normalised distance, $x/L$

Greek symbols

- $\delta$: thickness of the fin at the base
- $\theta$: normalised temperature, $(T - T_\infty)/(T_b - T_\infty)$
- $\theta_{s_n}$: normalised surface temperature at nth instant
- $\phi_d$: deposition rate, m$^2$·K/W·s
- $\phi_r$: removal rate, m$^2$·K/W·s
- $\eta_c$: cleanliness efficiency
- $\tau$: normalised time

1 Introduction

Fouling refers to any undesirable deposit on heat transfer surfaces which increases the resistance to both heat transfer and fluid flow. Frequently, fouling has a decisive effect on the initial design as well as the operation of heat transfer equipment. When the equipment uses finned surfaces, the effect of fouling on its performance could be expected to be significant. Of several kinds of fouling accumulation behaviour, asymptotic fouling is a common industrial occurrence.

Asymptotic fouling is of great practical importance, since it indicates the possibility that the heat exchanger can work for an extended time without additional fouling or without maintenance repairs. Reverse solubility salts are among foulants with asymptotic fouling. They often occur in cases of heating and evaporation, in which extended surfaces are employed to increase the heat flux. A complex situation is produced due to the combination of temperature variation along the fin axis, the sensitivity of reverse solubility fouling to local wall temperature and asymptotic fouling behaviour. In such a situation, the effect of fouling on the performance of finned heat transfer equipment is difficult to predict even qualitatively. The present analysis has the objective of throwing light on the mechanism and generating an analytical tool for aiding in the design in such situations.

2 Summary of background information

Fouling has long been considered as the major unresolved problem in heat transfer. Fouling literature abounds in
results of research into the various types of fouling, kinds of fouling behaviour, fouling mechanisms and empirical data on specific fouling streams.


In water desalination, Malik and Fareg [4] consider scale formation and fouling as facts of life to be reckoned with. Najibi et al. [5] have studied calcium sulphate scale formation during subcooled flow boiling. Only a few experimental investigations of heat transfer to electrolyte solutions can be found in the literature [6]. This deficiency appears to be due to the difficulties associated with fouling experiments: the long period over which fouling has to be monitored and measured, the need to perform these experiments in industrial equipment and the requirement that the experiments be carried out in steady flow with no interruptions during the entire period. Tube side fouling occurs even in flooded-evaporator-water-chillers as observed by Haider et al. [7]. Thus, waterside fouling due to reverse solubility and direct solubility electrolytes occurs in heating and in cooling.

Fouling is postulated to be the consequence of both deposition and removal processes, so that one might express the fouling rate as suggested by Kern and Seaton [8].

$$\frac{dR_t}{dt} = \phi_d - \phi_r \tag{1}$$

It has been suggested by Taborek et al. [9] that the deposition rate function may be expressed as

$$\phi_d = k_1 \exp\left(-E/RT_s\right) \tag{2}$$

and that the removal rate function may be of the form

$$\phi_r = k_2 R_t \tag{3}$$

so that

$$\left(\frac{dR_t}{dt}\right) = k_1 \exp\left(-E/RT_s\right) - k_2 R_t \tag{4}$$

Equation (4) can be integrated to give the fouling resistance as a function of time

$$R_t = K \exp\left(-E/RT_s\right) \left(1 - \exp\left(-\tau\right)\right) \tag{5}$$

where the asymptotic value of fouling resistance is given by

$$R_t^{\infty} = K \exp\left(-E/RT_s\right) \tag{6}$$

and hence

$$R_t = R_t^{\infty} \left[1 - \exp\left(-\tau\right)\right] \tag{7}$$

The asymptotic fouling resistance is attained when the deposition and removal rates become equal. $R_t^\infty$ is an important quantity because, if predicted, it can be used in analysis as well as design of heat exchangers for fouling service. Based on their investigations, Knudsen and Story [10] have reported a least squares fit for their data as

$$R_t^{\infty} = 4.7 \times 10^8 \exp\left(-10098.5/T_s\right) \tag{8}$$

These values will be used for representative calculations in this analysis. The validity of the analysis is, however, general and could use other empirical data. No information is available in the literature on the analysis of fouling on finned surfaces. The present work aims to fulfill this need and outlines how it might be used in the design of heat exchangers.

3 Mathematical development of the model

The assumptions made in the model are:

1. The analysed fin is thin and has constant conductivity, so that its low Biot number permits one-dimensional approach.
2. The characteristic time for heat penetration from fin base to tip, $L^2/\alpha$ is much smaller than the fouling time constant $t_c$. This is generally true of fins used in practice.
3. The convective heat transfer coefficient is invariant over the fin surface.
4. The thermal conductivity of the scale is constant throughout its thickness and is not affected by variation in temperature.
5. The fouling growth is asymptotic.
6. The boundary conditions are constant base temperature and insulated-tip.
7. The variation of surface temperature within a time step is negligible.

4 Formulation

The general governing equation is obtained with the assumption that the temperature of the fin does not vary with time unless it is subjected to fouling.

If $h$ represents the free stream convective heat transfer coefficient over the fin surface and the presence of the foulant adds to the convective resistance giving an apparent reduced external heat transfer coefficient, $h_s$, the resulting fin equation in normalized form is

$$A \frac{d^2\theta}{dx^2} + \left(\frac{dA}{dx}\right) \frac{d\theta}{dx} + h_s L \theta \left(\frac{dS}{dx}\right) = 0 \tag{9}$$

where $h_s = \frac{1}{R_t + R_f}$ and $R_c = \frac{1}{h}$

therefore $h_s = \frac{h}{1 + h R_f}$

$A$ and $S$ are functions of $X$, the normalized distance.

Defining $A = A_s Y(X)$ and $S = P_s L P(X)$

Equation (9) is written as

$$Y(X) \frac{d^2\theta}{dx^2} + \frac{dY(X)}{dx} \left(\frac{d\theta}{dx}\right) - G(X) \theta = 0 \tag{10}$$

where $G(X) = (m L)^2 \frac{dp}{dx} \frac{1}{1 + h R_f}$ and $m = \sqrt{\frac{h}{1 + h R_f}}$

For various configurations, the parameters are

1. Rectangular fin

$$Y(X) = 1 \quad P(X) = X \quad G(X) = \frac{(m L)}{1 + h R_f}$$