A numerical study of heat transfer of a porous block with the random porosity model in a channel flow

W. S. Fu, S. F. Chen

Abstract In this paper, heat transfer of a hot plate with a porous block in a channel flow is numerically investigated. A porous block is simulated as a fin type heat sink. The random/artificial porosity models are used to generate the distribution of porosity. In fact, the distribution of porosity in porous medium is irregular, thus the random porosity model is more realistic than the constant or variable porosity model to describe the phenomena happening in porous medium. Therefore, the distribution of porosity of porous block obeys the random porosity model, and the factors of mean porosity and standard deviation are taken into consideration. The variations of the porosity and the velocity in porous block are no longer smooth. For obtaining more heat transfer rate, the artificial porosity model is proposed. The heat transfer rates of the several cases derived by the artificial porosity model are better than those of the random porosity model. The thermal performance of porous block is larger than that of solid block as the mean porosity is larger than 0.5.

List of symbols

- $C_f$: specific heat of fluid, J kg$^{-1}$ K$^{-1}$
- $D_p$: dimensionless mean bead diameter, $d_p/h$
- $d_p$: mean bead diameter, m
- $F$: inertial factor
- $h$: dimensionless distance from the jet inlet to the top surface of the block
- $h_p$: dimensionless height of the block
- $h_s$: local heat transfer coefficient, W m$^{-2}$ K$^{-1}$
- $K$: permeability, m$^2$
- $k$: thermal conductivity, W m$^{-1}$ K$^{-1}$
- $l_p$: dimensionless length of the block, $l_p/h$
- $N(a, b^2)$: normal distribution with mean $a$ and standard deviation $b$
- $Nu$: local Nusselt number along the heated wall of the block
- $\overline{Nu}$: mean Nusselt number
- $P$: dimensionless pressure
- $Pr$: Prandtl number, $v/\alpha$
- $p$: pressure, N m$^{-2}$
- $Re$: Reynolds number, $u_0h/v$
- $T$: temperature, K
- $U$: dimensionless velocity in the X direction, $u/u_0$
- $u$: velocity in the x direction, m s$^{-1}$
- $u_0$: maximum inlet velocity, m s$^{-1}$
- $V$: dimensionless velocity in the Y direction, $v/u_0$
- $v$: velocity in the y direction, m s$^{-1}$
- $X, Y$: dimensionless Cartesian coordinates, $x/h, y/h$
- $x, y$: Cartesian coordinates, m
- $\bar{\mu}$: magnitude of velocity vector

Greek symbols

- $\alpha$: thermal diffusivity, m$^2$ s$^{-1}$
- $\bar{\epsilon}$: mean porosity
- $\mu$: viscosity, kg m$^{-1}$ s$^{-1}$
- $\nu$: kinematic viscosity, m$^2$ s$^{-1}$
- $\theta$: dimensionless temperature
- $\rho$: density, kg m$^{-3}$
- $\sigma_c$: standard deviation of porosity
- $\Phi$: computational variable
- $\psi$: dimensionless stream function

Superscripts

- $n$: the $n$th iteration index
- $\text{mean}$: mean value
- $\rightarrow$: velocity vector

Subscripts

- C.V.: control volume
- $e$: effective value
- $f$: external flow field
- $i$: index; inlet
- $p$: porous block
- $s$: solid block
- $w$: solid wall
- $X$: along the X direction

1 Introduction

Recently a pin fin heat sink which is an indispensable thermal dissipation device shown in Fig. 1a is widely applied in electrical cooling system. Due to the complex geometry of the pin fin heat sink, an appropriate method adopted to analyze the heat transfer characteristics of the pin fin heat sink has been hardly proposed. However, from a macropoint view, the pin fin heat sink can be approximately regarded as a kind of porous medium. Therefore,
that the results obtained from analyzing the porous medium simulate the results of the pin fin heat sink is worth studying.

For facilitating the analysis, the porosity of the porous medium was usually assumed as constant, called a constant porosity model. However, Roblee et al. [1] and Benenati and Brosilow [2] based on their experimental results observed the porosity varied significantly in the near wall region. Schwartz et al. [3] conducted experimental studies and measured the maximum velocity which is normally called channelling effect in the near wall region. These phenomena directly validated that the porosity regarded as a variable was more realistic. Cheng et al. [4] pointed out that in lot of literature the porosity was simulated as a damped oscillatory function of the distance from the wall and the damped oscillatory phenomenon was insignificant as the distance was larger than five bead diameters for packed beds. Therefore, the variation of the porosity is assumed as an exponential function of the distance from the solid wall and is called a variable porosity model. Based upon the above experiences, up to now two different models are adopted to derive the individual equations of fluid flow and heat transfer for the porous medium.

However, Georgiadis et al. [5–7] studied the unidirectional flow and heat transport phenomena in the random porous medium. The results indicated the mean flow rate $U(\varepsilon)$ based on the random porosity was larger than that $U(\bar{\varepsilon})$ based on the constant porosity when the Forchheimer model of flow was held. Saito et al. [8] studied the effects of the porosity and void distributions on the permeability by direct simulation Monte Carlo method and found that the permeability depended not only on the porosity but also on the void distribution strongly. These facts indicated that except for special screen process the sizes of the beads are extremely difficult to be uniform. In fact, the distribution of porous medium is difficult to obey the distributions of the constant or variable porosity model mentioned above. That the porosity in porous medium is irregular or random is more realistic, this distribution of porosity is conveniently named a random porosity model. Although the effect of the random porosity distribution on the flow field had been discussed in the past, however, most of previous studies only concentrated on special physical phenomena. Fu and Huang [9] studied flow and thermal fields of a porous block with the random porosity distribution under a laminar slot impinging jet. The results showed that the porosity near the heat wall should be smaller to enhance heat transfer rate for an impinging jet flow. Based on the results of the above literature, for obtaining more efficient performance of the use of porous medium, the distribution of porosity in porous medium need to be arranged appropriately. This special arrangement of the distribution of porosity in porous block is briefly called artificial porosity model. Besides, for expending the application of porous medium