BOOK REVIEW

Fundamentals of Semigroup Theory

by John M. Howie

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Communicated by Karl H. Hofmann

The general plan of this new book of John M. Howie coincides with that of his previous very successful book (see [1]), which has been used as a basis for the new monograph. As the author says in the Preface, most of the previous volume “has been substantially rewritten to provide the perspective that seems most relevant to contemporary research, and significant amounts of new (post 1976) material have been incorporated, especially in the exercises.” That made the book almost 30% longer than [1]. The titles of the chapters are the same, except that “regular semigroups” are added to the title of Chapter 2, Chapter 4 is now called “Completely regular semigroups” (rather than “Unions of groups”), Chapter 5 “Orthodox semigroups” became “Other classes of regular semigroups,” and a new chapter “Free semigroups” was added. The book is dedicated to the memory of A. H. Clifford.

This is the 13-th book on the general algebraic theory of semigroups (see [2] for the list of the previous twelve books). Since the previous volume [1] is so well known, I shall pay more attention to new material when describing the contents of the new book.

Chapter 1 “Introductory ideas” is very similar to Chapter 1 of [1]. Among the differences one can mention a theorem on the structure of rectangular bands and presentations of semigroups in section 1.6.

Comparing to [1], Chapter 2 “Green's equivalences; regular semigroups” has an additional section about Nambooripad's sandwich sets.

Chapter 3 “0-simple semigroups” is rewritten somewhat more extensively than the previous two chapters, completely simple semigroups (without 0) as well as isomorphisms of Rees matrix semigroups are considered in special sections now.

Chapter 4 “Completely regular semigroups” now has a section on varieties of such semigroups (considered as algebras with two operations, a binary multiplication and a unary inversion).

Chapter 5 “Inverse semigroups” is the longest in the book. It contains a new treatment of congruences via the kernel-trace pairs introduced by Petrich and stemming from Wagner [Vagner]. The section “Fundamental inverse semigroups” of [1] is now called “The Munn semigroup”.

*This semigroup was considered by Wagner in 1953, who mentioned some of its properties. Apparently, Munn was not familiar with this, when he reintroduced this semigroup. Munn's paper
There are two new sections on E-unitary inverse semigroups and free inverse monoids.

Chapter 6 “Other classes of regular semigroups” is extensively rewritten. It consists of three sections that cover locally inverse semigroups, orthodox semigroups, and semibands (that is, regular semigroups generated by their idempotents).

Chapter 7 “Free semigroups” is new. It has two sections on general properties of free semigroups and codes. The author says that his “intention is to convey the flavour and some of the methods” of this large and vital area.

Chapter 8 “Semigroup amalgams” is the final and second longest. It has been rewritten more extensively than many other parts of the book and has new sections on systems (equivalently, actions of monoids on sets, or representations of monoids by transformations of sets), direct limits, free extensions and free products, and on the extension property.

The author has moved historical remarks from the beginning of the chapters and sections to the end of each chapter and added many new exercises. The list of references is changed, many of the earlier works are no longer included. Altogether, the new list contains almost 90% more items than earlier. The list of symbols and index are expanded too.

The typographical quality of the book is high, although detecting where new sections begin is more difficult now than in the edition of 1976 because of the font and spacing used for the section titles. The only misprints I found were carried over from [1].

The previous volume [1] appeared at the time when books devoted to general semigroup theory were a rarity. It was preceded by only two other monographs written on this subject in English (by Clifford and Preston and by Petrich).

Now this situation has changed for the better, and there are many first-rate books on the general algebraic theory of semigroups, topological and analytic semigroups, special topics of the theory, as well as proceedings of numerous meetings devoted to semigroups. Everyone who, like the reviewer, appreciated [1] and used it for teaching would certainly want to use this new excellent book. It is addressed both to the specialist and non-specialist alike, as well as to graduate students. Many of its sections are quite accessible to undergraduate students interested in algebra. Each of the three books on the algebraic theory of semigroups published in English in recent years (by P. M. Higgins, P. A. Grillet, and J. M. Howie) complements the other two, because no single volume can reflect the diversity of semigroups as it exists now. This is a cause for optimism. The new book is a valuable contribution to the corpus of semigroup theory.

References


was seminal, and the semigroup turned out to be quite useful in inverse semigroup theory. Later Wagner returned to fundamental inverse semigroups, which he called “anti-groups,” building on his previous results.

**Counting the books written in Russian and Japanese by Ljapin, Tamura and Yamada, [1] was the sixth book on general semigroups.