Challenges to the Assessment of Time-to-Proof of Mathematical Conjectures

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In the 27 December 2010 issue of New Scientist, several articles discussed progress in predicting the timing of new discoveries and forecasting the future of science and technology. In particular, Samuel Arbesman and Rachel Courtland used the waiting times for solving 18 mathematical problems to estimate the probability that the “P versus NP problem” will be solved by 2024: they arrive at roughly 50% [1]. To do this, they constructed an approximate representation of the cumulative distribution of waiting times from formulation to proof (referred to as “times-to-proof” hereafter), based on this set of 18 solved mathematical problems.

Their methodology created a heated debate on the New Scientist website [2]. For instance, a comment posted on 3 February 2011, 10:27:11 GMT, criticized the authors’ methodology, stressing that “their method of estimation looked only at problems that actually were solved,” which may introduce a selection bias. A more formal attack was published in New Scientist on 2 February 2011 [3]. In a nutshell: using a probability distribution amounts to assuming that the underlying generating process is stationary, but stationarity may not hold over the decades and centuries corresponding to the investigated data, as the population of mathematicians has grown significantly and their theorem-proving technology has arguably improved due, e.g., to cumulative knowledge, computers, and collective work mediated by Internet and social network tools.

Nevertheless, we think the question posed by Arbesman and Courtland is interesting. Not only is it an attempt to guess when an unsolved problem such as the P versus NP conjecture might be settled, but it also raises the issue of the evolution of productivity of mathematics throughout history. In this spirit, we revisit this question and analyze a larger database of 144 conjectures including both closed and open conjectures.

But first, we assure you that we are well aware of the main caveats with attempting a statistical quantification of the generation of mathematical results during its (relatively recent) history.

First, any assessment of the time-to-proof distribution of mathematical conjectures can be criticized as being meaningless if it ignores their content and context as well as several other issues. Specifically,

1) A “time-to-proof” depends on the content of the conjecture. Related to the content is the question of whether mathematicians judge the conjecture worth pursuing, and why. This is not unique to mathematics. Indeed, consider a typical individual (a mathematician in our context) who is subjected to a flow of information...
(research papers to read, conferences she is attending, visits of colleagues, and so on) and requested tasks (teaching, administration, etc.), under time, energy, regulatory, social, and monetary constraints. She will respond by a sequence of actions that themselves contribute to the flow of influences spreading to other mathematicians. Remarkably, stationary distributions have been documented quantitatively for the waiting times between triggering factor and response of a number of human activities, such as the waiting times until an e-mail message is answered [4], the time intervals between consecutive e-mails sent by a single user and time delays for e-mail replies [5], the waiting time between receipt and response in the correspondence of Darwin and of Einstein [6], and the waiting times associated with web browsing, library visits, and stock trading [7]. In each of these activities, one could forcefully argue that the reported distributions may be meaningless, aggregating “carrots” and “potatoes,” because each single different human activity is strongly influenced by its specific content and the proximate interest it represents to its user. Yet, the evidence suggests a kind of universal behavior that is worth investigating, even for mathematical conjectures. Despite the variability in the characteristics of conjectures (and of other human problems and activities), there might be a homogenous process underlying the generation of the problems and their resolutions. In this spirit, although the underlying generating mechanism(s) of the purported distributions are not known for certain, we suggest that recent modeling progress based on priority queuing theory may be relevant [8, 9].

Another theoretical approach consists in thinking of mathematical research as a bundle of random walks in some high-dimensional mathematical space, such that intersections between them or crossing of some boundary corresponds to a successful outcome and the establishment of the proof. Such models have been argued to apply for instance to the space of investment strategies used by a large populations of traders, explaining the long memory of financial volatility as resulting from the statistical properties of random walk crossing in arbitrary spaces [10].

2) In addition, the definition of what constitutes a “proof” has changed with time, and, within a given definition paradigm, proofs hold different standings. For instance,

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