We introduce a shaper setup which takes advantage of the polarization Jones vector yielding the vector modulators employing \( \phi \). We demonstrate the experimental setup capable of phase, amplitude, and polarization control. Next, in Sect. 4 we show limited polarization forming with the double pass shaper. Afterwards, we present example pulses in Sect. 5 and then discuss the full polarization control in Sect. 6. The conclusions are derived in Sect. 7.

2 Theory of a double layer spatial light modulator

The molecules of the liquid crystals of commercial double-layer modulators are orientated 45° relative to the optical table plane for the first layer and −45° for the second [21]. One can regard nematic crystals as waveplates with a defined orientation of the optical axes and with a variable retardation. We will employ Jones matrices to show the influence of such a setup on a \( P \) polarized (the electrical field oscillates in the plane of the optical table, 0°) wave. By multiplying the matrices \( LC_{45°}[\phi_a] \) and \( LC_{-45°}[\phi_b] \), corresponding to rotated 45° and −45° waveplates with \( \phi_a \) and \( \phi_b \) as variable retardance between extraordinary and ordinary wave, the Jones matrix of a modulator can be obtained. Multiplying this matrix by the \( P \) polarization Jones vector yields the vector that represents polarization and phase after the modulator as a function of the retardation \( \phi_a \) and \( \phi_b \) of the two arrays

\[
LC_{-45°}[\phi_b] \times LC_{45°}[\phi_a] \times P = \frac{1}{2} \exp \left[ \frac{i}{2} (\phi_a + \phi_b) \right] \begin{bmatrix} \cos \left( \frac{1}{2} (\phi_a - \phi_b) \right) \\ i \sin \left( \frac{1}{2} (\phi_a - \phi_b) \right) \end{bmatrix} . \tag{1}
\]

Clearly, the phase can be directed by the sum of the retardances \( \phi_a + \phi_b \) and the polarization by the difference \( \phi_a - \phi_b \). Alternatively, a \( P \) polarizer placed after the second array transmits only the \( P \) component, and in this way, amplitude modulation is realized.

Considering the vector describing the polarization term in (1), it is obvious that a double-layer modulator is capable of generating a linear polarization only when \( \phi_a - \phi_b = n \ast \)
π; \( n \in \mathbb{Z} \). Therefore it is impossible to create any other linear polarization than \( P \) or \( S \) (\( S \) is \( 90^\circ \), the electrical field oscillates in the plane perpendicular to the optical table). All other retardances yield elliptical polarizations with the major axes of the ellipse orientated along the \( P \) or \( S \) direction, including the special cases \( \phi_d - \phi_b = \pi/2 + n \pi \) when the polarization is circular.

3 Serial setup

Since we showed in the previous section that a double array modulator is not capable of regulating the phase, amplitude and polarization simultaneously and independently, we suggest utilizing more than three arrays and a polarizer, for example two double-array modulators separated by a polarizer.

A practical and considerably less expensive solution is to use one modulator with arrays wide enough to arrange it in a two-pass configuration, as shown in Fig. 1. This is equivalent to employing four arrays and a polarizer in between the second and the third one. The amplitude is controlled in the first pass, and the polarization and phase in the second pass. The disadvantage of this solution is a lower resolution compared to a single pass shaper setup.

The Jones vector for the four arrays with the \( P \) orientated polarizer \( PP \) in between the second and the third array (the double-pass setup) is given by

\[
LC_{-45^\circ}[\phi_d] \times LC_{45^\circ}[\phi_i] \times PP \times LC_{-45^\circ}[\phi_b] \\
\times LC_{45^\circ}[\phi_c] \times P = \\
\cos \frac{1}{2}(\phi_d - \phi_b) \cdot \exp \left( \frac{1}{2} (\phi_d + \phi_b + \phi_c + \phi_a) \right) \\
\times \left( \cos \frac{1}{2}(\phi_c - \phi_d) + i \sin \frac{1}{2}(\phi_c - \phi_d) \right). \tag{2}
\]

The polarization term is again in the form \( \begin{pmatrix} A \\ iB \end{pmatrix} \) where \( A \) denotes the ellipse major axis orientated along the \( P \) direction and \( B \) along the \( S \) direction. This arrangement still does not allow to turn the orientation of the major axis. However, considering dipole transition moments in atoms and simple linear molecules, the polarization of photons required for transitions are \( P \), \( S \), or circular, and this is assured by the serial setup. Another matter connected with the double pass arrangement is the necessary pixel assignment of the first and second pass. In the case of a grating the different incidence angles correspond to a different angular dispersion which will affect the spatial distribution of the spectrum. Although for a small change of the angle, this effect should not be significant. We made a simulation of the grating dispersion to estimate the magnitude of this difference. By comparing spectral distributions on the first lens, one can estimate the potential errors of the pixel overlap. The calculated difference between the spectral distributions in Fig. 2 displays that along the bandwidth of 90 nm the spectral mismatch is 90 μm. Translating this to pixel width corresponds to illuminating 130 pixels in the first pass and 131 in the second, which is only a small misalignment. Also, the range of 90 nm is three times the FWHM of our spectrum and there is very low intensity on the side wings of the spectrum, so the pixel coupling is not critical there. To conclude, for a given grating and chosen angles, the mismatch of the pixel pairs can be neglected.

![FIGURE 1 Serial shaper configuration. The laser beam enters the first grating (density of 600 lines/mm) as the arrow indicates, and its spectral components (FWHM = 30 nm) are being spatially separated and collimated by a cylindrical lens (f = 250 mm). Then they are transmitted through the modulator (SLM-640 from CRi) located in the Fourier plane, a second lens and hit the second grating (density of 600 lines/mm). Afterwards the beam is sent through a polarizer and redirected again on the second grating at a different incident angle. The spectral components are sent through a different part of the modulator, reenter the first grating and leave the setup.](image1.png)

![FIGURE 2 Graph (a) shows the wavelength distribution on the modulator in the first and the second pass in the serial shaper setup. Graph (b) presents the difference of the wavelength distribution for the first and the second pass with the extracted difference for the central wavelength, so the curve passes zero at \( \lambda = 800 \) nm. The used laser spectrum is indicated on both graphs by a gray Gaussian function.](image2.png)