Two-dimensional dissimilar electromagnetic cloak for irregular regions

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Abstract A general method to design a 2D dissimilar cloak for irregular regions is presented by operating a nonlinear transformation in polar coordinates. The material parameters avoid discontinuities while the thickness of the cloak shell is effectively limited in elongated directions. Full-wave simulations of an elliptic cloak, a rectangular cloak, and an arbitrary-shape cloak are performed to verify the validity of the design. Both the material parameters and the scattering widths of different models are calculated and illustrated for comparison. This method provides a possible approach for designing complex shaped cloaks.

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1 Introduction

Recently, transformation optics, especially invisibility cloaks based on coordinate transformation, has received intense attention. The basic idea of this method is the equivalence between trajectories propagating through the warped space, which can be considered as the topological interpretation, and propagating through the special electromagnetic structures with complex constitutive relations, which provides the material interpretation [1, 2]. For two-dimensional (2D) circular cloaks, the validity of the approach was first confirmed computationally in geometric optic limits [2], then in full-wave finite element simulations [3], and furthermore, at microwave frequencies [4] and optical frequencies [5] with the help of metamaterial technology, although a reduced set of cloak parameters was used in the experimental demonstrations. Researches from other viewpoints, such as theoretical and practical problems, have been reported [6–10], and the transformation method was even introduced into acoustic applications [11].

Up to one year ago, most studies on cloak design were limited to circular cylindrical cases (2D) and spherical cases (3D) due to the symmetry in geometry and the simplicity in calculation. After that, the square cloak [12], eccentric elliptic cloak [13], and homocentric elliptic cloak [14–16] were proposed one after the other. Furthermore, designs for ellipsoids, rounded cuboids, and rounded cylinders were also reported [17]. The authors also proposed the design of 2D cloaks with arbitrary geometries [18]. Meanwhile, high-order transformations have been analytically studied [19–21]. However, for all the examples mentioned above, the inner and outer boundaries of the cloak device are geometrically similar.

For the convenience of practical applications, especially when cloaking elongated objects, the outer boundary of the cloak shell is usually desired to be not similar to the inner boundary. Kwon and Werner proposed a 2D elliptic cloak design having a uniform thickness by using the spatial transformation in a composite coordinate system [22]. The cloak, which is a union of two annular regions of uniform thicknesses, involves discontinuous constitutive parameters across the interface, and is therefore not easy to calculate or fabricate. Hu et al. discussed arbitrary shape transformation media from a deformation view and got noticeable results numerically [23], but the method still has difficulties in picking up material parameters to fabricate. The concept...
of 'reshaper' was proposed later [24], providing a general recipe for designing transformation media.

In this paper, we present a general method to design 2D dissimilar cloaks by operating a nonlinear transformation in the polar coordinate system. To show the simplicity and efficiency of this recipe, the method is applied to the elliptic cloak and approximate rectangular cloak, where a concept of superellipse is used in the latter example. An arbitrary shaped cloak is also displayed to show the generality. The designs avoid discontinuities in constitutive parameters. Furthermore, the total electric-field distributions are simulated by using the finite element method, and the scattering widths are calculated in order to quantitatively evaluate the cloaking performance. Cloaks of ideal and lossy models are compared according to the scattering width parameter.

## 2 Transformation equations

Consider an arbitrary continuous spatial transformation in the form

\[
\chi^\alpha(\chi^\alpha) = A_{\alpha}^{\alpha'} \chi^\alpha.
\]  

(1)

Here \( A_{\alpha}^{\alpha'} \) is the Jacobian transformation matrix, while the primed indices denote vectors in the transformed space. As tensor densities of weight +1, the associated permittivity and permeability transform as [2, 25]

\[
\varepsilon^{\prime ij} = \det(\Lambda^{\prime ij})^{-1} \Lambda^{\prime ij} \varepsilon^{ij},
\]

\[
\mu^{\prime ij} = \det(\Lambda^{\prime ij})^{-1} \Lambda^{\prime ij} \mu^{ij}.
\]

(2)

The absolute value signs of the determinant are dropped, as the transformations we discuss are warping and squeezing, where the determinants are always positive.

In polar coordinates, the boundary of an object to be cloaked, i.e. the inner boundary of the cloak shell, can be expressed as

\[
f_{\text{in}} = a \cdot \rho(\theta),
\]

(3)

where \( \rho(\theta) \) is the normalized polar equation obtained by setting \( \rho(0) = 1 \), and \( a \) can be comprehended as a radial scale factor. In the same way, suppose the outer boundary to be

\[
f_{\text{out}} = b \cdot \varphi(\theta).
\]

(4)

Since the cloaking transformations are conducted in the radial direction, to form a dissimilar cloak, we define a non-linear function

\[
r' = a + \frac{b \varphi(\theta) - a \rho(\theta)}{b \varphi(\theta)} \cdot r,
\]

(5)

where \( r \) and \( r' \) represent the radial scale factors normalized to \( \rho(\theta) \) in the original space and the transformed space, respectively. Meanwhile no operation is defined in the \( \theta \)- or \( \varphi \)-direction.

Obviously, (5) achieves results that take all the fields illuminating on the device and compress them into the region between \( f_{\text{in}}(\theta) \) and \( f_{\text{out}}(\theta) \), i.e. the cloak shell. In contrast to the transformations previously presented, (5) ensures different squeezing scales in different directions, which leads to unsymmetrical warping of the space (the topological interpretation) and more complex changing of the constitutive parameters (the material interpretation). For the special case that \( \rho(\theta) = \varphi(\theta) \), the transformation relation degenerates to a very simple form according to (5), which reveals the symmetrical squeezing scale in a similar cloak.

### 2.1 Elliptic cloak

For an ellipse with its semi-axes lying along \( x \)-, \( y \)-directions of length \( x_0 \) and \( y_0 \), respectively, the polar equation can be expressed as

\[
f_{\text{in}}(\theta) = \frac{x_0}{\sqrt{\cos^2 \theta + (x_0 \sin \theta/y_0)^2}}.
\]

(6)

Considering the geometric properties of ellipses, an ellipse as described in (7) can be very close to the geometry that is defined by giving a thickness increment \( \Delta \) to the ellipse in (6):

\[
f_{\text{out}}(\theta) = \frac{x_0 + \Delta}{\sqrt{\cos^2 \theta + [(x_0 + \Delta) \sin \theta/(y_0 + \Delta)]^2}}.
\]

(7)

A dissimilar elliptic cloak can be realized by substituting (5), (6), (7) into (2) [18]. The cloak device consists of only