Experimental laminar Rayleigh-Bénard convection in a cubical cavity at moderate Rayleigh and Prandtl numbers

J. Pallares, M. P. Arroyo, F. X. Grau, F. Giralt

Abstract Rayleigh-Bénard convection in a cubical cavity with adiabatic or conductive sidewalls is experimentally analyzed at moderate Rayleigh numbers ($Ra \leq 8 \times 10^4$) using silicone oil ($Pr = 130$) as the convecting fluid. Under these conditions the flow is steady and laminar. Three single-roll-type structures and an unstable toroidal roll have been observed inside the cavity with nearly adiabatic sidewalls. The sequence from the conductive state consists of a toroidal roll that evolves to a diagonally oriented single roll with increasing Rayleigh number. This diagonal roll, which is stabilized by the effect of the small but finite conductivity of the walls, shifts its axis of rotation towards to two opposite walls, and back to the diagonal orientation to allow for the increase in circulation that occurs as the Rayleigh number is further increased. Conduction at the sidewalls modifies this sequence in the sense that the two initial single rolls finally evolve into a four-roll structure. Once formed, this four-roll structure remains stable when decreasing the Rayleigh number until the initial single diagonally oriented roll is again recovered. The topology and the velocity fields of all structures, characterized with visualization and particle image velocimetry, respectively, are in good agreement with numerical results reported previously for the cavity with adiabatic walls, as well as with the numerical predictions obtained in the present study for perfectly conducting lateral walls.

Nomenclature

$A$ aspect ratio, e.g., $A_x = L_x/L_z$ and $A_y = L_y/L_z$
$C$ thermal conductance defined by Eq. (1)
$d$ wall thickness [m]
$g$ gravitational acceleration [m s$^{-2}$]
$k$ thermal conductivity [W m$^{-1}$ K$^{-1}$]
$L$ height of the cavity and reference length scale [m]
$Pr$ Prandtl number, $v/\nu$
$Ra$ Rayleigh number, $g\beta\Delta T L^3/\nu\kappa$
$t$ time [s]
$T$ temperature [K]
$u, v, w$ velocity components [m s$^{-1}$]
$x, y, z$ Cartesian coordinates [m]

Greek letters

$\alpha$ thermal diffusivity [m$^2$ s$^{-1}$]
$\beta$ thermal expansion coefficient [K$^{-1}$]
$\nu$ kinematic viscosity [m$^2$ s$^{-1}$]
$\Delta$ increment

Superscripts and subscripts

* dimensionless quantity
c critical value
f fluid
w sidewalls

1 Introduction
Flow instability in a fluid layer heated from below is an important fluid flow problem that has been extensively studied in the past because of its theoretical implications and practical applications. Rayleigh-Bénard (hereinafter RB) convection inside cavities is a convenient flow system to study the transition to turbulence (Busse 1978). Many engineering problems related to thermal comfort in buildings, crystal growth, and solar collectors depend on this type of natural convection. Reviews of this problem have been carried out by Normand et al. (1977), Oertel (1982), Yang (1988), Koschmieder (1993), and Mukutmoni (1994).

Laminar RB convection experiments in small-aspect-ratio parallelepipedic enclosures ranging from $1 \times 1.2 \times 2$ to $1 \times 2.1 \times 4$ have been carried out by Arroyo and Savirón (1992), Jäger (1982), Dubois and Bergé (1981), and Gollub and Benson (1980) for $0.71 \leq Pr \leq 1.30$. These studies show that the preferred fluid motion in the steady regime are convection rolls with their axis of rotation...
parallel to the shorter horizontal dimension of the cavity. The velocity field is approximately two-dimensional away from the ends of the rolls. Arroyo and Savirón (1992) provide quantitative information of the three-dimensional spatial properties of the steady counter-rotating roll pattern formed in the 1 × 1.2 × 2 enclosure as the Rayleigh number changes. These authors showed that the velocity field of the roll pattern is three-dimensional even for a Rayleigh number just above the critical value. Jäger (1982), Dubois and Bergé (1981), and Gollub and Benson (1980) analyzed time-dependent phenomena by imposing different numbers of steady rolls as initial conditions.

Natural convection in cubical cavities has been extensively studied because of its geometrical simplicity. The flow in a side-heated cubical cavity, which is driven by imposing a temperature gradient perpendicular to the gravity vector, invariably occurs with ascending and descending currents parallel and close to the heated and cooled vertical walls, respectively (Dabiri and Gharib 1996). The types of flow patterns or stable structures that develop in RB convection increase in number when the imposed temperature gradient is parallel to the gravity vector and the parallelepiped enclosures change towards a cubical geometry with no preferred horizontal direction. To our knowledge, RB convection in a cubical cavity with perfectly insulated sidewalls was first studied numerically by Aziz and Hellums (1967) with a stream function-vorticity formulation. They reported a diagonally oriented roll at \( Ra = 3500 \) and \( Pr = 1 \). Ozoe et al. (1976) obtained with the same formulation the diagonally aligned roll, as well as a single roll oriented with respect to two opposite sidewalls. The variation of the Nusselt number with Rayleigh number calculated by these authors at \( Pr = 1 \) shows at \( Nu = 1 \) a critical Rayleigh number of about \( 3.5 \times 10^5 \), which is in agreement with the \( Ra_c = 3446 \) predicted by Catton (1970) using linear stability analysis and the \( Ra_c = 3800 \) determined experimentally by Heitz and Westwater (1971) in a cubical cavity with nearly adiabatic lateral walls.

The numerical study carried out more recently by Hernández and Frederick (1994) showed the occurrence of a toroidal-type roll structure at \( Ra = 8 \times 10^3 \) and \( Pr = 0.71 \) for a cube with perfectly insulated lateral walls. Pallares et al. (1996a) and Pallares et al. (1999) completed the numerical studies for the cubical cavity mentioned above and extended the range of Rayleigh numbers to \( Ra \leq 6 \times 10^6 \) at \( Pr = 0.71 \), 10, and 130. These authors identified four different single-roll structures – one with the axis perpendicular to two opposite vertical walls, two elongated towards two opposite horizontal edges, and one with the axis oriented towards two diagonally opposite vertical edges – a toroidal roll, and two four-roll structures. Symmetry elements in the horizontal midplane and appropriate nomenclature for these seven structures are given in Fig. 1. Pallares et al. (1999) also characterized numerically the flow transitions between these different topologies as a function of the Rayleigh number at each of the three Prandtl numbers studied.

The purpose of the present study is to characterize experimentally the RB flow patterns that develop inside a cubical cavity with sidewalls approaching the ideal cases of adiabatic or perfectly conducting boundaries. The topology and velocity fields of all structures developing inside a \( 12.5 \times 12.5 \times 12.5 \)-mm cavity filled with silicon oil (\( Pr = 130 \)) are determined with visualization and particle image velocimetry (PIV) in the range \( 5 \times 10^3 \leq Ra \leq 8 \times 10^4 \). Section 2 describes the experimental setup and the techniques used for velocity measurements. The complete discussion of the experimental results for cavities with adiabatic and conductive lateral walls is reported in Sect. 3.

2 Experimental

2.1 RB cells

The experimental setup used in this work is the one developed by Arroyo and Savirón (1992). Figure 2 shows the vertical cross sections of the two RB cells used. The two cubical cells \( (L_x = L_y = L_z = 12.5 \text{ mm}) \) were constructed between two 4-mm-thick copper plates, which were easily sandwiched between two 10-mm-thick copper blocks. The blocks and the plates were joined with semiconductor paste to allow for good thermal contact. Maximum horizontal misalignments of the cavities were estimated to be ±0.1 degrees.

The two 10-mm-thick copper blocks were kept at constant temperature (±0.01 °C) by circulating water from two independent thermostatic baths. Four Chromel-Alumel thermocouples connected in series and embedded in the copper blocks measured the temperature difference \( \Delta T \) between the bottom and top plates of the cavities. The potential difference produced by this system (163.2 µV/°C) was measured by a digital microvoltmeter.

The material and thickness of the sidewalls assembled between the copper plates to form the cavities fixed the two lateral thermal boundary conditions imposed. Perfectly adiabatic sidewalls cause all heat to be transferred...