Utility-gap dominances and inequality orderings

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Abstract The justification for using Lorenz dominance as an inequality ranking condition has been based on the aggregate social welfare comparison and the Pigou–Dalton principle of transfers. Since both the aggregating aspect of the social welfare function and certain implications of the principle of transfers are debatable, ordering conditions stronger than Lorenz dominance are worth exploring. A particularly interesting direction to pursue is to follow the frequently invoked notion that inequality is the “gap” between the rich and the poor. This paper follows this notion to formally propose a unified utility-gap concept and characterizes several utility-gap based conditions as general stronger-than-Lorenz-dominance ranking criteria. Specifically, we propose utility-gap dominance which requires all pairwise utility-gaps in one distribution to be uniformly smaller than those of the other distribution. We then explore a conceptually weaker dominance concept – quasi dominance – which imposes conditions only on the gap between each person’s utility and some reference utility point of the distribution.

1 Introduction

The measurement of income inequality concerns the degree of dispersion of income among the recipients; inequality exists as long as not all incomes are the same or there are gaps between the rich and the poor. Among the numerous measures of inequality, Lorenz dominance is probably the best known and the most widely applied measure. In his seminal paper, Atkinson (1970) justified the use of Lorenz dominance...
dominance based on comparing the aggregate social welfare of each distribution – with the same mean income the Lorenz-dominating distribution has more aggregate social welfare than the one that is Lorenz-dominated. A key assumption in Atkinson’s justification is that social welfare is aggregated and the agent who compares social welfare is the so-called “social planner” who is not a part of the distributions being compared. It follows that situations may arise in which one distribution is judged to be more equal than another distribution in aggregation even if some members of the distributions perceive the dispersion of income somewhat differently. Since the intensity of income inequality ultimately concerns people in the distribution rather than the hypothetical “social planner” per se, it may be useful to explore some alternative and possibly stronger ordering criteria that are based on individual experiences in the distribution.

The necessity of searching for inequality measures stronger than Lorenz dominance also stems from some arguable implications of the Pigou–Dalton principle of transfers. As noted in the literature (e.g., Moyes 1994) the reduction in inequality resulted from a transfer of income from the rich to the poor is not always easy to perceive and to justify. Consider, for example, a three-person income distribution (2, 3, 7) and a transfer of $1 from the richest person to someone below. If the income is transferred to the poorest, then the new distribution (3, 3, 6) can be intuitively justified to have less inequality since everyone feels closer to each other in income. On the other hand, if the income is given to the person in the middle, it is not straightforward to justify the new distribution (2, 4, 6) to be more equal than the original one. This is because person 1 may feel he is now further away from person 2 although his gap to person 3 is narrowed; he may feel his gap to the rest of the population remains the same if his gap is represented by the aggregated dollar distance to other people, or enlarged if his sense of the gap is represented by an increasing and strictly concave utility function. If the inequality of the distribution is an aggregation of each individual’s perception and if person 1’s perception is given an enough larger weight than others’, then it is possible that (2, 4, 6) can be judged to be less equal than (2, 3, 7). In a two-person world, however, such an ambiguity may never arise because the gap between the two persons always shrinks no matter whether the gap is measured by income or by a strictly concave utility function. It follows that the principle of transfers is very reasonable in a two-person world but may require additional stipulations in a larger population. In fact, Dalton (1920) seemed to be aware of this potential controversy as he emphasized the two-person aspect of the set-up when he initially introduced the principle.

The purpose of this paper is to follow the notion of utility gap in interpersonal comparisons and to explore its implications on inequality rankings of income distributions. In the process, we introduce and characterize new partial ordering conditions that are stronger than Lorenz dominance in the sense that the new conditions imply Lorenz dominance but not vice versa. Note that this effort does not mean to abandon the Pigou–Dalton principle of transfers in its entirety; rather we seek to limit the applicability of the principle to some more restrictive forms of the transfers such as the one transferring income from the richest person to the poorest person. It follows that we are not interested in dominance conditions that

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1 This can also be the case even from the point of view of an impartial social planner. In Amiel and Cowell’s (1992) questionnaire studies, 64% of the survey participants (viewed as social planners) do not think that (1, 4, 7, 10, 13) is more unequally distributed than (1, 5, 6, 10, 13).