Ranking intersecting Lorenz curves

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Abstract This paper is concerned with the problem of ranking Lorenz curves in situations where the Lorenz curves intersect and no unambiguous ranking can be attained without introducing weaker ranking criteria than first-degree Lorenz dominance. To deal with such situations two alternative sequences of nested dominance criteria between Lorenz curves are introduced. At the limit the systems of dominance criteria appear to depend solely on the income share of either the worst-off or the best-off income recipient. This result suggests two alternative strategies for increasing the number of Lorenz curves that can be strictly ordered; one that places more emphasis on changes that occur in the lower part of the income distribution and the other that places more emphasis on changes that occur in the upper part of the income distribution. Both strategies turn out to depart from the Gini coefficient; one requires higher degree of downside and the other higher degree of upside inequality aversion than what is exhibited by the Gini coefficient. Furthermore, it is demonstrated that the sequences of dominance criteria characterize two separate systems of nested sub-families of inequality measures and thus provide a method for identifying the least restrictive social preferences required to reach an unambiguous ranking of a given set of Lorenz curves. Moreover, it is demonstrated that the introduction of successively more general transfer principles than the Pigou–Dalton principle of transfers forms a helpful basis for judging the normative significance of higher degrees of Lorenz dominance. The dominance results for Lorenz curves do also apply to generalized...
Lorenz curves and thus provide convenient characterizations of the corresponding social welfare orderings.

1 Introduction

In empirical analyses of income distributions it is common practice to make separate comparisons of mean incomes and Lorenz curves. The Lorenz curve, which was introduced as a representation of inequality, is concerned with income shares without taking account of differences in mean incomes. By displaying the deviation of each individual income share from the income share that corresponds to perfect equality, the Lorenz curve captures the essential descriptive features of the concept of inequality. Thus, adopting the Lorenz curve as a basis for judging between income distributions means that we focus solely on distributional aspects. The widespread use of the Lorenz curve in applied work shows that focusing on distributional aspects is of interest in its own right, irrespective of how we judge between level of mean income and degree of inequality in cases where they conflict. For welfare judgments about the trade-off between mean income and inequality we refer to Shorrocks (1983), Ebert (1987) and Lambert (1985, 1993a).

Ranking Lorenz curves in accordance with first-degree Lorenz dominance means that the higher of non-intersecting Lorenz curves is preferred. The normative significance of this criterion follows from the fact that the higher of two non-intersecting Lorenz curves can be obtained from the lower Lorenz curve by means of rank-preserving income transfers from richer to poorer individuals, which means that the criterion of first-degree Lorenz dominance is consistent with the Pigou–Dalton principle of transfers. Thus, when one Lorenz curve lies above another the higher Lorenz curve displays less inequality than the lower Lorenz curve. However, since Lorenz curves may intersect, which is often the case in applied economics, other ranking criteria than first-degree Lorenz dominance are needed to reach an unambiguous conclusion.

The standard practice for ranking intersecting Lorenz curves is to apply summary measures of inequality. However, as it may be difficult to find a single measure that gains a wide degree of support, it is of interest to search for alternative ranking criteria that are stronger than single measures of inequality and weaker than first-degree Lorenz dominance. To this end two alternative dominance criteria emerge as natural candidates; one that aggregates the Lorenz curve from below (second-degree upward Lorenz dominance) and the other that aggregates the Lorenz curve from above (second-degree downward Lorenz dominance). Since first-degree Lorenz dominance implies second-degree upward as well as downward Lorenz dominance we have that both methods preserve first-degree Lorenz dominance and thus are consistent with the Pigou–Dalton principle of transfers. However, the transfer sensitivity of these criteria differ in the sense that second-degree upward Lorenz dominance place more emphasis on transfers occurring in the lower rather than in the upper part of the income distribution,

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1 See e.g. Atkinson et al. (1995) who make cross-country comparisons of Lorenz curves allowing for differences between countries in level of income and Lambert (1993b) for a discussion of applying the criterion of first-degree Lorenz dominance as a basis for evaluating distributional effects of tax reforms.