The Strong No Show Paradoxes are a common flaw in Condorcet voting correspondences

Joaquín Pérez

Departamento de Fundamentos de Economía e Historia Económica, Universidad de Alcalá, Plaza de la Victoria, 28802 Alcalá de Henares (Madrid) Spain
(e-mail: joaquin.perez@uah.es)

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Abstract. The No Show Paradox (there is a voter who would rather not vote) is known to affect every Condorcet voting function. This paper analyses two strong versions of this paradox in the context of Condorcet voting correspondences. The first says that there is a voter whose favorite candidate loses the election if she votes honestly, but gets elected if she abstains. The second says that there is a voter whose least preferred candidate gets elected if she votes honestly, but loses the election if she abstains. All Condorcet correspondences satisfying some weak domination properties are shown to be affected by these strong forms of the paradox. On the other hand, with the exception of the Simpson-Cramer Minmax and the Young rule, all the Condorcet correspondences that (to the best of our knowledge) are proposed in the literature suffer from these two paradoxes.

1 Introduction

In the theory of voting, the prospects of finding a best voting method have been disappointing, due to the negative results obtained through the systematic axiomatic analysis employed during the last half of this century, including the Arrow Impossibility Theorem and the Gibbard-Satterthwaite result, with its subsequent developments and refinements. We know now that no voting method simultaneously fulfills some minimal properties that apparently are required by any reasonable method, that is to say, no method is free from paradoxes (failures to satisfy some intuitively compelling properties).

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However, it still makes sense to analyze and compare methods in order to select a reasonable one for a given setting. In this task of confronting methods and choosing the right one, perhaps the two main families are the Condorcet and the Positional families.

The interest and relevance of Condorcet voting methods stem from their fidelity to the democratic principle which asserts that if there exists a candidate that is favored by a majority of voters (in a face to face comparison) over any other, this candidate should be the only one chosen. This is called the Condorcet property. For the definition and analysis of the best known Condorcet methods, see Fishburn (1977), Tideman (1987), Laffond et al. (1995), and Peris and Subiza (1999).

The Positional or Scoring methods, and in particular the Borda method, aggregate the preferences of voters through a scoring technique which in some way extracts a measure of the intensity of these preferences. These methods have a normatively appealing consistency property (if two electorates are combined, the global result is coherent with the partial results). For the definition and analysis of the Positional and other related methods, see Young (1974, 1975) and Smith (1973).

Young and Levinglick (1978) have established the incompatibility of the Condorcet and consistency properties. A similar, but independent, property called Participation (none of the voters is disillusioned by submitting his true ballot) has also been shown in Moulin (1988) to be incompatible with the Condorcet property. Hence, every Condorcet method suffers from what has been termed as the No Show Paradox.

This paper, which follows Moulin (1988) and extends some results from Pérez (1995), explores the incidence in Condorcet voting correspondences of two symmetrical strong forms of the paradox (from now on called Strong No Show Paradox, or SNSP for short). In one of these forms, to be called Positive SNSP, there is a voter $V_1$ whose favorite candidate loses the election if $V_1$ votes honestly, but wins if $V_1$ abstains. In the other, Negative SNSP, there is a voter $V_1$ whose least preferred candidate wins if $V_1$ votes honestly, but loses if $V_1$ abstains.

Although not all Condorcet methods suffer from these paradoxes, the Simpson-Cramer Minmax method (free from Positive and Negative SNSP) and the Young method (free from Negative SNSP) are, as far as I know, the only exceptions among those proposed in the literature.

Section 2 presents the basic terminology and some known results. Section 3 defines the Positive and Negative SNSP, and identifies some weak domination properties that imply the paradoxes. Section 4 analyses, with the help of these properties (whenever possible), which known correspondences abide by the paradoxes, and Sect. 5 concludes the paper with some additional remarks, including the analysis of some other versions of SNSP.

2 Terminology

The terminology of Fishburn (1977) and Laffond et al. (1995) will be used whenever possible, with few modifications.