Condorcet choice correspondences for weak tournaments

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Abstract. Tournaments are complete and asymmetric binary relations. This type of binary relation rules out the possibility of ties or indifferences which are quite common in other contexts. In this work we generalize, from a normative point of view, some important tournament solutions to the context in which ties are possible.

1. Introduction

A tournament over a finite set of outcomes $A$ (candidates, decisions, ...) is a complete and asymmetric binary relation $T$ on $A$, where $aTb$ is interpreted as “alternative $a$ beats alternative $b$.” This kind of binary relation appears in many different models: sports competitions, biometric and psychometric models, collective choices (majority voting rules), etc. (see Moon 1968; Moulin 1986).

If there exists an alternative which beats all others (a Condorcet winner), then it is obvious that such an alternative must be selected. But this is not usually the case, and it is generally not clear which one (or ones) should be considered the winner of the tournament. Indeed, social choice theorists have...
formulated several choice rules, to tackle, in different ways, the difficulty posed by the nonexistence of a clear winner.

From a normative point of view, a great number of solutions have been proposed for the problem of choosing from a tournament: Copeland (1951); Slater (1961); top cycle (Schwartz, 1972); uncovered set (Fishburn 1977; Miller 1980); minimal covering (Dutta 1988); equilibrium set (Schwartz 1990); etc. Moreover, from a positive point of view, other solution concepts have been introduced with regard to choosing from a tournament: Banks set (Banks 1985); bipartisan set (Laffond et al. 1993); matching solutions (Levchenkov 1994). Laffond et al. (1995) provide a good set-theoretical comparison of the main solutions.

As Moulin (1986) points out, “a widely open question is the generalization to any complete relation on \( A \) (not necessarily asymmetric: indifferences are allowed)”. In most of the models, there is an actual possibility of ties: two football teams may tie; two candidates or alternatives may obtain the same number of votes;…

The aim of this paper is to generalize, from a normative point of view, some of the previously mentioned solutions for tournaments, to the context in which ties are allowed. We specifically analyze the top cycle, the uncovered set and the minimal covering for complete (not necessarily asymmetric) binary relations \( R \) (weak tournaments), in such a way that, when \( R \) is a tournament, the definition of such extensions coincides with the usual one.

In the existing literature, there are some papers that deal with what we have called weak-tournaments. In Schwartz (1986), two extensions of the top cycle are defined. In Bordes (1983), and Banks and Bordes (1988), different extensions of the notion of the uncovered set and the Banks set are introduced. Henriet (1985) extends the Copeland set. In Schwartz (1990) some proposals for extending the equilibrium set are presented. Finally, in a recent paper, Dutta and Laslier (1997) defined the essential set as an extension of the bipartisan set. From the axiomatic point of view (the one in which we are interested), it must be emphasized that Henriet (1985) provides characterizations of the Copeland choice rule in terms of neutrality, monotonicity and a new property called “independence of cycles”. Some of the other extensions we have mentioned are axiomatically analyzed, although not completely characterized.

In most of the above-mentioned extensions there appears to be no single clear-cut way of extending the tournament solutions to the case of weak-tournaments (a fact that has prompted some authors to propose different extensions). A similar problem occurs with some of the axioms used in the axiomatic characterizations: there are several possibilities of extending them into the context of weak-tournaments (as, for instance, in Banks and Bordes (1988), Condorcet consistence is generalized in three different ways: inclusive Condorcet, exclusive Condorcet and strict Condorcet). In our extensions, we have tried to maintain those properties (axioms) which are satisfied by the corresponding solutions in tournaments. Some of the axioms, of course, have had to be modified, and other new axioms have been introduced.