Preliminary phi-divergence test estimators for linear restrictions in a logistic regression model

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Abstract The problem of estimation of the parameters in a logistic regression model is considered under multicollinearity situation when it is suspected that the parameter of the logistic regression model may be restricted to a subspace. We study the properties of the preliminary test based on the minimum $\phi$-divergence estimator as well as in the $\phi$-divergence test statistic. The minimum $\phi$-divergence estimator is a natural extension of the maximum likelihood estimator and the $\phi$-divergence test statistics is a family of the test statistics for testing the hypothesis that the regression coefficients may be restricted to a subspace.

Keywords Logistic regression model · Phi-divergence test statistics · Minimum phi-divergence estimator · General linear hypotheses · Preliminary $\phi$-divergence test estimator

1 Introduction

Let $Y_1, \ldots, Y_I$, be independent binomial random variables with parameters $n_i$ and $\pi_i$, $i = 1, \ldots, I$. We denote by $n_{i1}$ the number of successes out of the $n_i$ subjects in the ith binomial random variable considered. In many cases, a series of nonstochastic explanatory variables $x_{i0}, \ldots, x_{ik}$, may be associated with each $Y_i$, $i = 1, \ldots, I$. We shall
assume that the binomial parameter, \( \pi_i \), is linked to the linear predictor \( \sum_{j=0}^{k} \beta_j x_{ij} \) via the logit function, i.e., \( \logit(\pi_i) = \sum_{j=0}^{k} \beta_j x_{ij} \), where \( \logit(p) = \log \left( \frac{p}{1-p} \right) \).

In the following we shall denote the binomial parameter \( \pi_i \), by \( \pi_i \equiv \pi(x_i^T \beta) \), where \( x_i^T = (x_{i0}, \ldots, x_{ik}) \), \( x_{i0} = 1, i = 1, \ldots, I \), and \( \beta = (\beta_0, \ldots, \beta_k)^T \) is a \((k+1) \times 1\) vector of unknown parameters with \( \beta_i \in (-\infty, \infty) \). The explanatory “design” matrix is \( X = (x_1, \ldots, x_I)^T \) and its rank is \( k+1 \). For more details about logistic regression models see Liu and Agresti (2005).

The usual way to estimate the parameters in the logistic regression model is using the Maximum Likelihood Estimator (MLE), however Pardo et al. (2005) considered the family of minimum \( \phi \)-divergence estimators, as a extension of the MLE, for estimating \( \beta \) and they found, through simulation, some interesting alternative estimators to the MLE in the sense of the mean squared error. We denote the minimum \( \phi \)-divergence estimator by \( \hat{\beta}_\phi \), and in the following we refer to it as the unrestricted minimum \( \phi \)-divergence estimator. Sometimes, we have some additional information about \( \beta \) given by a linear combination of the parameters in the way \( K^T \beta = m \). We shall denote by \( \hat{\beta}_\phi^{H_0} \) the minimum \( \phi \)-divergence estimator of \( \beta \) under the restrictions \( K^T \beta = m \).

Let \( S_N \) be a test statistic (likelihood ratio test, Pearson’s test, Wald test, etc.) for testing the compatibility of the restricted, \( \hat{\beta}_\phi \), and the unrestricted, \( \hat{\beta}_\phi \), minimum \( \phi \)-divergence estimators. If we accept the compatibility between them, we can use \( \hat{\beta}_\phi^{H_0} \) and if we reject the compatibility between them we shall use \( \hat{\beta}_\phi \). As a consequence of this testing followed by estimation procedure, estimation is dependent on a preliminary test of significance and the following preliminary test estimator results,

\[
\hat{\beta}_\phi^{\text{Pre}} = \hat{\beta}_\phi^{H_0} I_0(0,c)(S_N) + \hat{\beta}_\phi I_{1,c}(S_N),
\]

where \( I_A(y) \) denotes an indicator function taking the value 1 if \( y \in A \) and 0 if \( y \notin A \) and \( c \) is the critical point in the previous test statistic.

The seminal works about preliminary test estimators are in the papers of Bancroft (1944, 1964). The books of Judge and Bock (1978) and Saleh (2006) contain most of the important results in this area.

In Sect. 2 we present some basic concepts and results. In Sect. 3, we obtain the asymptotic distribution of \( \hat{\beta}_\phi^{\text{Pre}} \) under contiguous alternative hypotheses. Section 4 is devoted to get the asymptotic bias as well as the asymptotic distributional quadratic risk of \( \hat{\beta}_\phi^{H_0} \) and \( \hat{\beta}_\phi^{\text{Pre}} \). The performance of \( \hat{\beta}_\phi, \hat{\beta}_\phi^{H_0} \) and \( \hat{\beta}_\phi^{\text{Pre}} \) under \( H_0 \) as well as under contiguous alternative hypotheses is given in Sect. 5. In Sect. 6 we present a simulation study and finally in Sect. 7 some conclusions about the theoretical results obtained in Sect. 5.

## 2 Background and notation

We denote by

\[
\hat{p} = \left( \frac{n_{11}}{N}, \frac{n_{12}}{N}, \frac{n_{21}}{N}, \frac{n_{22}}{N}, \ldots, \frac{n_{11}}{N}, \frac{n_{12}}{N} \right)^T,
\]

the sample proportions in the logistic regression model, where \( N = \sum_{j=1}^{I} n_j \), \( n_{i2} = n_i - n_{i1} \), while the corresponding probabilities by