Conditionally stochastic domination of generalized order statistics from two samples

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Abstract In this note, we investigate less restrictive conditions on the model parameters which enable one to establish the likelihood ratio and the usual stochastic orderings of conditional generalized order statistics from two samples. The main results strengthen and generalize the corresponding results established recently in the literature.

Keywords Usual stochastic order · Hazard rate order · Likelihood ratio order

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1 Introduction

Let \( n \in \mathbb{N} = \{1, 2, \ldots\}, k > 0 \) and \( \bar{m} = (m_1, \ldots, m_{n-1}) \in \mathbb{N}^{n-1} \) be parameters such that
\[ \gamma_{r,n} = k + \sum_{j=r}^{n-1} (m_j + 1) > 0, \quad r = 1, \ldots, n - 1, \]  

(1.1)

if \( n \geq 2 \) (\( \tilde{m} \) arbitrary if \( n = 1 \)). If the random variables \( U_{(r,n,\tilde{m},k)} \), \( r = 1, \ldots, n \), possess a joint density of the form

\[
f_{U_{(1,n,\tilde{m},k)}, \ldots, U_{(n,n,\tilde{m},k)}}(u_1, \ldots, u_n) = \left( \prod_{j=1}^{n-1} \gamma_{j,n} \right) \left( \prod_{i=1}^{n} (1 - u_i)^m_i \right) (1 - u_n)^{k-1}
\]

for all \( 0 \leq u_1 \leq u_2 \leq \cdots \leq u_n < 1 \) with \( \gamma_{n,n} = k \), then they are called uniform generalized order statistics (GOSs). Now, let \( F \) be an arbitrary distribution function. The random variables

\[
X_{(r,n,\tilde{m},k)} = F^{-1}(U_{(r,n,\tilde{m},k)}), \quad r = 1, \ldots, n,
\]

(1.2)

are called the GOSs based on \( F \), where \( F^{-1} \) is the right continuous version of its inverse. In the particular case \( m_1 = \cdots = m_{n-1} = m \), \( X_{(r,n,\tilde{m},k)} \) is denoted by \( X_{(r,n,m,k)} \) for \( r = 1, \ldots, n \). For more details on GOSs, one may refer to Kamps (1995a,b).

The concept of GOSs is a unification of several models of random variables arranged in ascending order of magnitude. In the past decade, properties of GOSs have attracted considerable attention in the literature; see Belzunce et al. (2005); Cramer and Kamps (2001, 2003); Franco et al. (2002); Hu and Zhuang (2005a,b, 2006); Khaledi and Kochar (2005), among others.

Let \( \{X_{(r,n,m,k)}, r = 1, \ldots, n\} \) and \( \{Y_{(r,n,m,k)}, r = 1, \ldots, n\} \) be the GOSs based on distribution functions \( F \) and \( G \), respectively. Denote by \( \lambda_F(x) \) and \( \lambda_G(x) \) the hazard rate functions of \( F \) and \( G \), and denote by \( [Z|A] \) any random variable whose distribution is the conditional distribution of \( Z \) given event \( A \). Hu et al. (2007) proved, among other things, that

- If \( m \geq -1 \) and \( k > 0 \), and if \( F \leq_{hr} G \), then

\[
[X_{(s,n,m,k)} - y \mid X_{(r,n,m,k)} > y] \leq_{st} [Y_{(s,n,m,k)} - y \mid Y_{(r,n,m,k)} > y]
\]

(1.3)

for \( y \in \mathbb{R} \) whenever \( 1 \leq r < s \leq n \);

- If

\[
F \leq_{lr} G, \quad k \geq 1 \text{ for } m \geq 0,
\]

(1.4)

or if

\[
F \leq_{hr} G, \quad k > 0 \text{ and } \frac{\lambda_G(x)}{\lambda_F(x)} \text{ is increasing in } x \text{ for } m \geq -1,
\]

(1.5)