Precedence tests and Lehmann alternatives

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Precedence tests are considered in the context of four classes of one-sided Lehmann-type alternatives: \( G = F^k (k > 1) \); \( G = 1 - (1-F)^k (k < 1) \); \( G = F^k (k < 1) \); and \( G = 1 - (1-F)^k (k > 1) \), where \( F \) and \( G \) are two continuous cumulative distribution functions. If an optimal precedence test (one with the maximal power) is determined for one of these four classes, the optimal tests for the other classes of alternatives can be derived. Application of this is given using the results of Lin and Sukhatme (1992) who derived the best precedence test for testing the null hypothesis that the lifetimes of two types of items on test have the same distribution. The test has maximum power for fixed \( k \) in the class of alternatives \( G = 1 - (1-F)^k \), with \( k < 1 \). Best precedence tests for the other three classes of Lehmann-type alternatives are derived using their results. Finally, a comparison of precedence tests with Wilcoxon’s two-sample test is presented.

Key Words: Best precedence tests; nonparametric; power; proportional hazards alternatives; semi-parametric; Wilcoxon two-sample test.


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1. Introduction

The one-sided two-sample problem is one of the basic problems in statistical testing. Assume that two independent samples $X_1, \ldots, X_m$ and $Y_1, \ldots, Y_n$ are given from continuous cumulative distribution functions $F$ and $G$, respectively. We wish to test the null hypothesis $H_0: F(x) = G(x)$ for all $x$, against the alternative hypothesis $H_a: G(x) \leq F(x)$ for all $x$, with strict inequality sign for at least one value of $x$. Many nonparametric tests for the two-sample problem are considered in the literature. In this paper only precedence tests are considered. General description and properties of precedence tests, as well as an overview of the literature are given in Chakraborti and van der Laan (1996 and 1997) for complete data and for censored data, respectively. In Section 2 a short overview of Lehmann alternatives and proportional hazards alternatives is given. The power of precedence tests is given in Section 3. In Section 4 it is proved that the power of a precedence test against four classes of Lehmann alternatives or proportional hazards alternatives can be derived if the power against one class of alternatives is given. Finally, in Section 5 a comparison of precedence tests with Wilcoxon’s two-sample test is given.

2. Some classes of Lehmann-type alternatives

If, in general, we wish to test the null hypothesis $H_0$ in the two-sample problem, alternatives that are considered often are the so-called shift alternatives. Nonparametric tests are compared with each other, and with their parametric counterparts by means of power against shift alternatives. We are thus led to examine the power of nonparametric tests. Against parametric shift alternatives, the power of a nonparametric test depends on the form of the underlying distribution. So we can compute the power, for example, under the assumption that the underlying distribution is normal. There is nothing contradictory about this because we examine power in order to determine how much we can stand to loose in power by using a nonparametric test, compared with a classical parametric counterpart, if the parametric model assumptions are really valid. However, for many distribution functions, the computation of the power function of many nonparametric tests has been a challenging problem. Partly to alleviate this problem, Lehmann (1953) introduced and proposed to use the so-called Lehmann alternatives in power calculations of two-sample rank tests.

Suppose that the order statistics of the $X$-observations are denoted by $X_{(1)} < \ldots < X_{(m)}$ and the order statistics of the $Y$-observations by $Y_{(1)} < \ldots < Y_{(n)}$. The ranks of the $X$'s and $Y$'s in the combined sample are denoted by $R_1 < \ldots < R_m$ and $S_1 < \ldots < S_n$, respectively. According to Lehmann (1953), suppose that the continuous $F$ and $G$ are related as $G = \Psi(F)$, where $\Psi$ is a continuous