Shape from shading based on needle map and cellular automata

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Abstract This paper presents a method for computing depth from a single grayscale image in two steps. Firstly, surface normals are parallelly and gradually adjusted by a procedure which includes three constraints: the smooth constraint ensures the recovered normals are smooth and integrable, the intensity gradient constraint ensures the recovered normals are consistent with the image gradient field, and the intensity constraint guarantees the recovered intensity is equal to the input image. Unlike their usage in global methods, those constraints are separately used in local area in our method. Secondly, the surface is recovered from needle map using a two-dimensional cellular automata system. An experimental assessment is provided for our methods on both real world images and synthetic images with known ground truth. The experiment results demonstrate this approach is practicable and has better precision than traditional methods.

Keywords Computer vision · Shape from shading · Needle map · Quadric smooth · Cellular automata

1 Introduction and related works

It is well known that shape from shading (SFS) is a typical computer vision method for surface reconstruction [8, 21]. Shading plays an important role in human perception of surface shape and the psychological research indicates that it is available to recover shape from a single image [10]. The goal of SFS is to derive the model of an existing free-form surface from only an image which is generated from this surface, and it deals with shape recovery from gradual variation of shading in the image. If the surface model can be obtained from a single digital image, the result is ambiguous because the inclination of a surface patch is determined by two directions (the slopes in x and y directions), while only one observation, namely the gray value corresponding to the patch, is available. So it is an ill-posed problem [7, 8, 21]. But we can overcome this well-known ambiguity by suggesting some various constraints or offering boundary information.

One way to overcome this problem is to use specific constraints. According to the constraints and computational methods, SFS techniques can be divided into two groups: global approaches and local approaches. Lee and Kuo [11] proposed a global approach to recover depth information using the intensity and the smooth constraints. Daniel and Durou [4] presented a global algorithm using a cost function including additional constraints. A typical cost function that Horn [7] had used involves three constraints including an intensity constraint, a regularization constraint and an integrability constraint. But Daniel and Durou did not use the quadric regularization term, instead, they required the gradients of the reconstructed image to be close to the gradients of the input image. All these methods are based on minimizing an energy function using variation calculus. A fundamental obstacle to progress in energy minimization approaches of SFS is the difficulty to develop a cost function which is uniquely minimized by a surface that closely matches the true sur-
face. So the nature of minimization method is hindered by searching for such an energy function. Some researchers tried to solve this problem by other methods instead of minimization one. Falcone and Sagona [6], E. Prados et al. [13] converted the energy function to Hamilton–Jaccobi type and then used viscosity solutions to solve it, but the computation was still complicated.

Tsai and Shah’s method [15] is a classic local approach to the SFS problem. They applied the discrete approximation for the height gradient using finite difference and linearized the irradiance equation, and so it is a purely local method but sensitive to noise in the image.

Recently, S. Collings et al. [3] used a piecewise quadric approach to solve the SFS problem. This method is non-variational and it creates a quadric surface to approximate the true surface in each local area, so it is necessary to solve nonlinear equations. Despite using Newton’s method, conjugate gradient or gradient descent, it is crucial and difficult to find a good initial guess for solving the nonlinear equations except the singular point.

Another problem in SFS is how to care the boundary conditions [9]. At the boundary the light rays just graze the surface of the object and it causes insufficiency of intensity information. Most methods use occluding boundary information to supply boundary conditions separately, so the algorithms become more difficult to implement.

Worthington and Hancock [18] proposed a method to solve the SFS problem based on surface normals. And they used curvature information to impose topographic constraints except intensity constraints. However, their method can not ensure the integrability of the surface and not directly attain the height of the surface.

The reconstructed results can be expressed in several ways: surface height, surface normal, surface gradient, and surface slant and tilt. Some methods [18, 19] use the normal at each point as variable in the computational process and the results are the normal corresponding to the input image, that is, needle map. In comparisons with surface height, surface normal is an intermediate result and it is not explicit, so some researchers are still concerned about how to recover surface height from needle map [14].

Aiming at those problems proposed above, we constitute a novel SFS framework which includes two stages. In the first stage where the needle map is recovered from input image, our method borrows some constraints used in global techniques but operates them in each local region simultaneously, so it enforces the needle map to satisfy these constraints and it’s more convenient to be implemented. This procedure contains three constraints: the intensity constraint as a hard constraint, region quadric patch fitting as a smooth constraint and an intensity gradient constraint which ensures the intensity gradient of the image reconstructed from needle map matches the gradient of the input image. At the mean time an operation mask containing boundary information is added to the algorithm and so each point can be treated equally. And in the second stage, a two-dimensional cellular automata system is applied for recovering surface height from needle map.

2 SFS framework

2.1 Notation

Suppose our image data are given on a discrete $m \times n$ grid. In general, a pixel will be labeled with $p(x, y)$, where $1 \leq x \leq m$ and $1 \leq y \leq n$, and both assume integer values only. But in most conditions, the SFS problem is not specifically given the domain $\Omega \in \mathbb{R}^2$, so we may choose to reconstruct the image in whatever domain which offers the boundary information. For an example illustrated in Fig. 1, the input image is in the $m \times n$ domain, and the boundary information for surface reconstruction is. For each pixel $p(x, y) \in \Omega$ we define the pixel neighborhood $N(p)$ to be the set of 4 pixels which are nearest to $p(x, y)$, and more explicitly it can be defined as $N(p) = \{(x + i, y + j) : i, j \in \{-1, 0, 1\} \text{ and } ij = 0 \}$. And an operation mask containing boundary information corresponding to the four-neighbor area is defined as $M(p)$, namely, the 4 neighbors of point $p(x, y)$.

In the computational process the results of reconstructed surface are denoted by the normal $n = (u, v, w)^T$ at each point.

2.2 Reflectance model and typical constraints

The central idea of SFS is that local regions in a grayscale image $I(x, y)$ correspond to illuminated patches of a continuous surface which can be described as $z(x, y)$. The intensity of input image varies depending on the material properties of the surface, the direction of the light source, and the direction of the surface normal. The imaging model for expressing the relationship between surface and image intensity is specified through a proper reflectance map. Most traditional SFS algorithms used three

![Fig. 1a, b. The input image and the reconstructed domain](image-url)