Wet Refractivity Tomography with an Improved Kalman-Filter Method

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ABSTRACT

An improved retrieval method, which uses the solution with a Gaussian constraint as the initial state variables for the Kalman Filtering (KF) method, was developed to retrieve the wet refractivity profiles from slant wet delays (SWD) extracted by the double-differenced (DD) GPS method. The accuracy of the GPS-derived SWDs is also tested in this study against the measurements of a water vapor radiometer (WVR) and a weather model. It is concluded that the GPS-derived SWDs have similar accuracy to those measured with WVR and are much higher in quality than those derived from the weather model used. The developed method is used to retrieve the 3D wet refractivity distribution in the Hong Kong region. The retrieved profiles agree well with the radiosonde observations, with a difference of about 4 mm km⁻¹ in the low levels. The accurate profiles obtained with this method are applicable in a number of meteorological applications.

Key words: wet refractivity, tomography, GPS, kalman filter


1. Introduction

Ground-based GPS meteorology can determine precipitable water vapor (PWV) with an accuracy of 1–2 mm (Bevis et al., 1992; Rocken et al., 1993), and has been used in a number of institutions and weather services (Fang, 2001; Wolfe and Gutman, 2001; Gendt et al., 2001). The determined PWVs are, however, not very easy to use in meteorological analysis and weather modeling because they cannot describe the vertical distribution of water vapor. Much research has been conducted to tackle this problem. The common approach is to extract the slant wet delay (SWD) from a GPS station to each satellite being tracked, and then construct a three-dimensional water vapor field or water vapor vertical profile at each GPS station using the tomography technique. With the technique, the atmosphere over the study area is divided into 3D cells (voxels) and the functional relationships between SWDs and the atmospheric state variables in the voxels are established. The state variables are then solved for. However, the direct solution of these unknown variables is difficult because of rank deficiency or ill-conditioning in the equations, which is due to the poor geometry of the satellite constellation or the unevenly distributed and relatively sparse ground GPS sites (Šeko et al., 2000). A few approaches have been proposed. Flores et al. (2000) and Braun et al. (2001, 2003) introduced additional horizontal and vertical constraints. Grandinart et al. (2004) used a Kalman-filter method, MacDonald et al. (2002) developed a 3-dimensional variational (3DVAR) method incorporating a weather model. These approaches have their own limitations. The method of additional constraints, if not based on real information, is less reliable, for the outcomes will greatly depend on the selection of the constraints. The Kalman-filter method relies on the initial values of the state variables to some extent. The 3DVAR is practical but complicated because of the involvement of a weather model. All these shortcomings have motivated this investigation. In our study, we have developed a method based on the Kalman-filter approach. The initial values of the state variables in the filter are obtained by using the solution with a loose Gaussian constraint. This method does not need additional constraints, like a vertical profile, or a background weather model, and it is easy to implement. The method has a great potential to monitor

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the 3-dimensional water vapor field for a region with a dense ground-based GPS array.

The second section in this paper gives a brief discussion on the methodology to derive SWDs with GPS. Since the quality of GPS-derived SWDs is crucial in the construction of a 3D water vapor field, it is checked in this study with other techniques. The ray tracing method to calculate SWDs with a weather model is also discussed. The developed method to construct a 3D water vapor field or a vertical profile with the tomography technique is discussed in section 3. Section 4 describes the experiments, followed by the analysis and discussion of the results. Some remarks based on the experiments are finally given.

2. Measuring SWDs with GPS

There are basically two approaches to gauge SWDs with GPS measurements: one is based on precise point positioning (PPP), and the other is based on the double difference of carrier phase measurements. The former directly measures SWDs, but needs the information on precise satellite orbits and clock errors; while the latter eliminates or largely reduces several source errors, but the data at a remote GPS station (say a few hundred or, better, thousand kilometers away) are required (Duan et al., 1996). In this study, we use the latter approach.

In the GPS double-differenced method, the slant wet delay can be finally expressed as:

$$ S(\lambda, \theta) = Z \cdot m_w(\theta) + N(\theta, \lambda) + \varepsilon, $$

where $\lambda$ and $\theta$ are the azimuth and elevation angles of a GPS satellite, respectively, $Z$ is the zenith wet delay, $N$ is the non-isotropic delay caused by the gradient, $m_w(\theta)$ is the wet mapping function, and $\varepsilon$ is the one-way residual which can be obtained from double differences of the residuals by imposing two assumptions (Aller et al., 2000). Moreover, $N$ can be expressed as (Bar-Sever and Kroger, 1998)

$$ N = m_g(\theta) \cdot (G_{18} \cos \lambda + G_{19} \sin \lambda), $$

with $G_{18}$ and $G_{19}$ being the gradients along the north-south and east-west directions, respectively, $m_g(\theta)$ being the mapping function for the gradients, which can be written as, $m_g(\theta) = 1/(\sin \theta \cdot \tan \theta + C)$, and $C=0.003$ (Chen and Herring, 1997).

To test the SWDs derived with the above method, we compared GPS-derived SWDs with those obtained with other techniques, viz., water vapor radiometer (WVR) and weather model. The SWDs can be measured with WVR directly and calculated from a weather model using the ray tracing method. The ray tracing method obtains SWDs from a numerical weather prediction (NWP) model with the following expression:

$$ S = 10^{-6} \int_L N_{\text{wet}} ds = 10^{-6} \int_L \left( k_1 \frac{e}{T} + k_2 \frac{e}{T^2} \right) ds, $$

where $T$ is absolute temperature, $e$ water vapor pressure, $N_{\text{wet}}$ the wet (non-hydrostatic) refractivity, $k_1$ and $k_2$ constants related to the refractivity, $L$ the path from the GPS receiver to a satellite, and $s$ the distance along the path. A signal path is determined by the three-dimensional coordinates of the points where the signal path intersects the full levels of the weather model. The intersection points for a signal path are calculated starting from the receiver and proceeding upwards using the ray tracing method (Treuhaft and Lanyi, 1987).

3. Recovery of water vapor field with tomography technique

Once the GPS data have been analyzed to determine SWD from each station to each visible satellite, the tomography technique can be used to recover the 3D water vapor field. Figure 1 illustrates the basic concept of the tomography method. The atmosphere is divided into voxels and the signal delay of each ray segment within each voxel can be calculated. Let $x_i$ be the wet refractivity at voxel $i$ of the tomographic domain. Then the total delay for ray $k$ is

$$ \sum_i c_{ik} x_i = S_k, $$

where $c_{ik}$ is a coefficient. The above equation can be rewritten in a matrix form as

$$ c_k^T x = S_k, $$

where $x$ is an unknown vector of wet refractivity for all voxels, and $c_k^T$ is the transposition of the $c_k$, $c_k^T = (c_{1k}, c_{2k}, \ldots, c_{mk})$. If there are $m$ rays in the area of study, they can be expressed as

$$ Cx = \mathbf{1}, $$

where $C$ is the matrix of coefficients with dimension $m \times n$. In general, the rank of $C$ is smaller than $n$ and the tomographic equations cannot be solved directly. This is because there are not enough GPS receivers in the region. Additional information is needed to obtain the solution. We have developed a method based on Kalman filtering, called the improved Kalman-filter method, as discussed below.