Approximations of the Scattering Phase Functions of Particles

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ABSTRACT

Based on anomalous diffraction theory and the modified Rayleigh-Debye approximation, a physically realistic model in bridging form is described to approximate the scattering phase function of particles. When compared with the exact method, the bridging technique reported here provides a reasonable approximation to the Mie results over a broader range of angles and size parameters, and it demonstrates the advantage of being computationally economic. In addition, the new phase function model can be essentially extended to other shapes and conveniently used in more complicated scattering and emission problems related to the solutions of the radiative transfer equations.

Key words: small light scattering, particles, phase function, bridging technique

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1. Introduction

The scattering phase function is one of the basic inputs in various radiative transfer models that describes the normalized angular distribution of scattered radiative energy and also represents the probability for radiation propagating from a given direction to be scattered into an elementary solid angle about another direction. In principle, the phase function is determined by solutions of the Maxwell equations for the interaction between the radiation field and particulate medium, and its quantitative calculation can be performed accurately by means of miscellaneous analytical and numerical techniques aimed at the electromagnetic scattering problems. These methods, including the separation of variables method, integral equation method, T-matrix method, point matching method, superposition method, finite element method and finite difference time domain method, have been recently reviewed by Wriedt (1998), Mishchenko et al. (2000), Liou (2002), and Kahnert (2003). However, the comprehensive investigation of solutions to electromagnetic scattering problems has been confined to a few simple shapes. Generally for the complex shaped particles, the common trick is employing the equivalent sphere model so as to make the Lorenz-Mie theory available. As has been pointed out, even with this simplifying assumption, strong angular oscillations and considerable computation time in the calculation may occur (Modest, 2003), which will enormously complicate the analysis of the radiative transfer at a given wavelength. This inconvenience has led to the design of a simple but accurate approximate phase function.

So far, quite a few models have been developed to approximate the scattering phase function. Heney and Greenstein (1941) proposed an empirical model (the so-called HG phase function) to describe the scattering of radiation in a galaxy. This expression, with $g$ (the asymmetric factor) as a single free parameter, has been widely used in atmospheric sciences because of its simple and analytic form. However, it can become highly inaccurate for some values of the particle size parameter and refractive index. To improve the precision of the HG phase function, some modifications and extensions have been suggested, which include the modified HG phase function (Cornette and Shanks, 1992; Draine, 2003), the two-parameter phase function (Reynolds and McCormick, 1980), and the three-parameter phase function (Irvine, 1965; Kat-tawar, 1975). Besides the HG-type approximations, there are other valuable forms which have been proposed, e.g., by Chu and Churchill (1955), McKel-

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and Caldas and Semnão (2001). However, the abovementioned phase functions cannot describe the asym-
ptotic limit for small and large particles simultaneously.

In this paper, we present a new scattering phase
function valid for various sizes by the bridging tech-
tique. In section 2, the asymptotic behavior of the
phase function formula is discussed and a new bridg-
ing function for it is developed. Section 3 contains
comparisons of the new approximation to the exact
method for spheres. The main results are summarized
in section 4.

2. Development of the phase function formulas

Consider the general scattering problem of an arbi-
trarily shaped particle characterized by volume $V$ and
projected area $A$. As is well known, the scattering
properties of particles that are optically small enough
can be represented by the Rayleigh-Debye approxima-
tion (RDA), and those that are optically large enough
can be approximated by anomalous diffraction theory
(ADT). The primary object of this article is to com-
brine the two asymptotic approximations into a general
expression that is capable of describing the phase func-
tion for particles of all sizes.

2.1 Small particle limit

The RDA, otherwise known as the Rayleigh-Gans
approximation or Born approximation (Irvine, 1965),
as a powerful tool, is widely applied to the problems
of light scattering by small particles. General con-
tions of the validity of the RDA are $kd < 1$ and $kd - 1 < 1$, $d$
represents the characteristic particle size, $m$ is the complex index of refraction of particle relative to the medium and $k$ is the wave
number. These conditions imply that the particles are
assumed to be not too large compared to the wave-
length of radiation (although they may be larger than
in the case of Rayleigh scattering) and optically "soft".

The fundamental assumption of the RDA is that each
volume element of the scattering object is excited only
by the incident field, and the electric field inside the
scatterers is equal to the incident field. This simplif-
ied assumption leads to significant analytical progress
in many specific cases. On the other hand, some im-
provements and extensions for RDA have been made
already (Acquista, 1976; Kiblestov, 1984; Kiblestov
and Mehlin, 1991; Kiblestov et al., 1991; Muinonen,
1996). If the particle irradiated by unpolarized light
is assumed homogeneous and isotropic, the scattering
phase function in the small particle limit, $p(\theta)_{\text{small}}$ can
be expressed as

$$p(\theta)_{\text{small}} = a |b_0|^2 (1 + \cos^2 \theta),$$

where $\theta$ is the scattering angle, $a$ is the normalization
constant, and $b_0$ is the form factor, which is given by

$$b_0 = \frac{1}{V} \int \exp[i(k_i - k_s) \cdot r'] d^3 r',$$

with $i = \sqrt{-1}$ is the imaginary unit, $k_i$ and $k_s$ are
wave-vectors of the incident field and scattering field,
respectively, $d^3 r'$ is the volume element at the point
$r'(x', y', z')$ within scatterer. However, Shimizu (1983)
pointed out that Eq. (2) does not yield the correct an-
gular position for the extrema in the scattering curves.
Saxon (1955) and Gordon (1955) discussed and sug-
gested respectively a modified RDA (MRDA) method,
which allows the refractive index of the particle to en-
ter the calculation, whereas in the unmodified RDA
the scattering results are independent of $m$. Unfor-
unately, the MRDA scheme is not exact enough for
particles comparable in size to wavelength, so here we
develop a new scheme to improve the original MRDA
and rewrite Eq. (1) as

$$p(\theta)_{\text{small}} = a_0 |b_0|^2 (1 - t)(|b_2| + \gamma)^2 (1 + \cos^2 \theta),$$

with

$$t = \frac{b_2}{b_0} = \frac{1}{V} \int \exp[i(m(k_i - k_s) \cdot r') d^3 r']$$

$$\gamma = \frac{b_2}{b_0} = \frac{1}{V} \int \exp[-c_1 x_{\text{vrp}}^3] dx_{\text{vrp}}$$

In Eq. (3), $c_1 = 5Re([m(1 - i)]/8), Re$ represents
the real part of a complex quantity $x_{\text{vrp}} = 3kV/(4P)$ is the
equivalent-sphere size parameter. Then new normal-
ized factor $a_0$ is determined according to relation:

$$\int p(\theta)_{\text{small}} d\Omega = 1,$$

where $d\Omega = \sin \theta d\theta d\phi$ is the element of solid angle
and the integration is over all scattering angles.

2.2 Large particle limit

The traditional ADT is a widely used Eikonal-type
approximation (Van de Hulst, 1957; Chen, 1984) and
was initially developed to calculate the extinction and
absorption cross section for large optically soft spheres.
Xu and Alfano (2003) put a statistical interpretation
on it recently. ADT assumes that the index of refraction is close to unity and that the size parameter is
large enough. This assumption implies that the refra-
tion and the reflection are negligible as the ray pases through the particles, and it allows simple analy-
tical expressions for many geometrical shapes. These
consist of spheres (Van de Hulst, 1967), spheroids
(Greeble and Melzer, 1990; Fournier and Evans,
1991), ellipsoids (Streekstra et al., 1994), cubes (Nap-
per, 1967; Maslowska et al., 1994), prismatic columns