Yield stress of structured fluids measured by squeeze flow

Abstract Various structured fluids were placed between the parallel circular plates of a squeeze-flow rheometer and squeezed by a force $F$ until the fluid thickness $h$ was stationary. Fluid thickness down to a few microns could be measured. Most fluids showed two kinds of dependence of $F$ on $h$ according to an experimentally-determined thickness $h^*$. If $h > h^*$ then $F$ varied in proportion to $h^{-1}$ as predicted by Scott (1931) for a fluid with a shear yield stress $\tau_0$. The magnitude of $\tau_0$ from squeeze-flow data in this region was compared with the yield stress measured by the vane method. For some fluids $\tau_0$ measured by squeeze flow was less than the yield stress, suggesting that the yield stress of fluid in contact with the plates was less than the bulk yield stress. If $h < h^*$ then $F$ varied approximately as $h^{-5/2}$ and the squeeze-flow data in this region analysed with Scott’s relationship gave a yield stress which increased as the fluid thickness decreased. This previously unreported effect may result from unconnected regions of large yield stress in the fluid of size similar to $h^*$ which are not sensed by the vane and which become effective in squeeze flow only when $h < h^*$.

Key words Squeeze flow · Yield stress · Structured fluids · Soft solids · Rheometry

Introduction

Structured fluids or soft solids such as pastes or particulate suspensions usually possess a shear yield stress $\tau_0$. Knowledge of $\tau_0$ allows the deformation of materials to be predicted and modelled. Barnes (1999) has reviewed the concept and the measurement of yield stress for soft solids and structured fluids and Nguyen and Boger (1992) have described yield stress measurement methods. The need to understand friction at interfaces and the effect of boundary conditions on complex flow of yield-stress fluids was emphasised by Piau (1998). The deformation of a fluid squeezed between two parallel plates has a long history in rheometry (Hoffner et al. 1997; Yang 1998; Barnes 1999). For complex fluids and soft solids it is still being studied (Adams et al. 1998a; Adams et al. 1998b; Denn 1998; Denn and Marrucci 1999). Such deformation is common in many materials of practical utility, ranging from foods (Campanella and Peleg 1987; Hoffner et al. 1997) to electro- and magneto-rheological fluids (Rankin et al. 1998; El Wahed et al. 1999; See et al. 1999).

In the work described here a number of common structured fluids were squeezed between parallel circular plates by a force $F$ until the plates became stationary at a fluid thickness $h$, when the absence of strain rate ensured that viscous contributions to $F$ were absent. The relationship of Scott (1931), modified by Sherwood and Durban (1996), was used to analyse the experimental data. The fluids ranged in yield stress between about 10 to 100 Pa which allowed the vane method (Keentok 1982; Dzuy and Boger 1985; Alderman et al. 1991; Nguyen and Boger 1992) to be used to obtain comparative yield stress data. Few previous workers appear to have used stationary squeeze flow to measure the yield stress of fluids or soft solids. Of these, only Dukes (1957), Covey and Stanmore (1981), and Campanella and Peleg (1987) seem to have compared their results.
with another method. In our apparatus (Fig. 1) the force \( F \) was derived from weights; an engineer’s dial gauge kept the plates parallel and measured their separation down to a few microns, over which distance large-scale structure may occur in complex colloidal fluids (Piau 1998).

**Theory**

Squeeze-flow theory and expressions used to analyse experimental results are reviewed here. Except where stated we assume the squeezed fluid to be a cylinder of constant diameter \( D \) with a thickness \( \theta \) equal to the plate separation. In our apparatus (see Fig. 1) the diameter of the top plate 8 was taken to define \( D \). Apart from Eq. (1), the squeeze force expressions below refer to plates in the limit as \( \theta \to 0 \).

Newtonian glycerol was used to check the apparatus and method. For a Newtonian liquid of viscosity \( \eta \) the squeeze force \( F \) is given by the Stefan-Reynolds relation (Phan-Thien et al. 1987)

\[
F = \frac{3\pi \eta D^4}{32\theta^3} \dot{\theta}
\]  

where \( \dot{\theta} \equiv d\theta/dt \) (\( \theta \) is time), and slip of fluid at the plate is absent. The effect of slip on the Stefan-Reynolds relation was investigated by Laun et al. (1999). From Eq. (1) \( \theta \) varies as \( \theta^3 \) for a constant force \( F \) such that \( \theta \to 0 \) only when \( \theta \to 0 \).

Covey and Stanmore (1981) considered the squeeze flow of the Bingham and the Herschel-Bulkley yield stress fluid, assuming in both cases a non-slip boundary condition in a lubrication analysis (pressure balanced by shear stress and \( \theta/D \ll 1 \)). For these fluids in the case when \( \theta \to 0 \) it is found that

\[
F = \frac{\pi D^3 \tau_0}{12 \theta}
\]  

(2)

where \( \tau_0 \) is the Bingham or the Herschel-Bulkley yield stress. This relation was given originally by Scott (1931) who revised it erroneously, as described by Covey and Stanmore. Adams et al. (1994) compared plasticity and lubrication analyses of the uniaxial compression of plastic materials in the absence of slip. The plasticity analysis applied to a rigid-plastic solid gave

\[
F = \frac{\pi D^3 \sigma_0}{4} \left( 1 + \frac{\mu D}{3h} \right)
\]  

(3)

where \( \mu \) is the coefficient of friction at the sample-plate boundary. Adams et al. took \( \sigma_0 = \sigma_0/\sqrt{3} \) for a von Mises solid, giving \( \mu = 1/\sqrt{3} \), so Eq. (3) reduces to Eq. (2) in the limit as \( \theta \to 0 \). The lubrication analysis applied to a Herschel-Bulkley fluid in the limit as \( \theta \to 0 \) gave

\[
F = \frac{\pi D^3 \tau_0}{4} \left( 1 + \frac{D \tau_0}{3h\sigma_0} \right)
\]  

(4)

which again reduces to Eq. (2) if \( \theta/D \to 0 \).

Sherwood and Durban (1996) analysed the squeeze flow of a rigid-plastic solid and a power-law fluid. In the case of the rigid-plastic solid the shear stress at the plate surfaces was taken to be a fixed fraction \( m \) of the yield stress \( k \), and in the case of the power-law fluid to be a fixed fraction \( m \) of the effective von Mises stress. For the rigid-plastic solid for all \( \theta \), and for the Bingham fluid if \( \theta \to 0 \), they obtained

\[
F = \frac{\pi D^3 m \tau_0}{12 \theta} + \frac{\sqrt{3} \pi D^2 \tau_0}{8} \left( \sqrt{1 - m^2} + m^{-1} \arcsin m \right)
\]  

(5)

where we have put \( k = \tau_0 \) (Hill 1950). If \( mD \gg h \) Eq. (5) becomes

\[
F = \frac{\pi D^3 m \tau_0}{12 \theta}
\]  

(6)

i.e. the Scott relation (Eq. (2)) with \( \tau_0 \) replaced by \( m \tau_0 \).

Sherwood and Durban (1998) also analysed the squeeze flow of a Herschel-Bulkley fluid in which they used the same fluid-plate boundary condition as for the rigid-plastic solid and the Bingham fluid. For the case of \( mD \gg h \) and \( \theta \to 0 \) their result for \( F \) (Eq. 30) reduces again to Eq. (6).

It is interesting to consider the behaviour of a squeeze flow experiment in the case of practically frictionless contact between the plates and the adjacent fluid, i.e. \( m \to 0 \). For the rigid-plastic solid and the Bingham fluid Eq. (5) gives

Fig. 1 Schematic diagram of the squeeze-flow rheometer. See the text for a detailed description.