On the Extreme Lunar Velocities

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1. Introduction: A historical account

The problem addressed in this paper is an accurate determination of the apparent extreme angular velocities of the Moon as observed from the Earth.1

As our historical records show in East and West alike, Man has constantly been busy with the most remarkable, daily behaviour of our closest celestial neighbor with its greatest apparent motion against the background of the stars. Different from the Sun’s daily motion which is all the year round nearly uniform, the markedly variable lunar velocities were easily perceptible from even very crude ancient observations. However, the actual, quantitative recognition that the Moon’s motion is not constant, came, according to the natural principles of astronomy, necessarily long after the Moon’s periodic relation to the Sun, the synodic period, had been firmly established. This historical process seems to be best followed in China.

Since the discovery of the Metonic cycle at around −590 (Yabuuti, 279f.), this mighty astronomical tool provided the fundamental, periodic functions of the Chinese luni-solar calendars. On the basis of this cycle and the year length the Chinese determined the mean length of the synodic month by computation (Tab. A, line 13). As the year length, measured by solstitial observations, decreased from the rough and convenient value, 365 1/4 days (Tab. A, 1-e), to that celebrated value of 365.2425 days (Thung Thien calendar, +1199–1207), the mean length of the synodic month came frequently close, or even equivalent, to those accurate values of Babylonia, Tycho Brahe and modern times [e.g.

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1 In the years 1989–90 I was asked, independently of one another, by two distinguished historians of astronomy, Lis Brack-Bernsen and David King, about the Moon’s extreme velocities. This was my starting point of the present problem. Ch. 3 was worked out in 1990 and Chs. 4 and 5 in 1998–2000. With a new Introduction containing a short historical account the long delayed paper is now ready to be published. I am grateful for the assistance of Dr. E. Künn and Dr. F. Charette.
29.5305954 ... (San Chi calendar, +384–517), also Tab. A, 5-f], and this particularly so after the Metonic cycle was replaced by more accurate luni-solar cycles as adopted from + 412 onwards by nearly all calendars [e.g. 29.5305915 ... (Thai Ming calendar, +510–589, and many others); cf. Tab. A, 7-, 11-, 12-f; for the above discussion cf. Yabuuti, 82, 281f., esp. 340ff.]. This development holds true also for the mean lunar velocity determined by computation (Tab. A, 5-b).

The earliest record of an awareness of the Chinese that the Moon's velocity is variable dates from the first century B.C. (Tab. A, 2-c, -d), and more than a century later we hear that this primitive observation was refuted as erroneous (3-c, -d). In the second century A.D. the variable lunar velocities, expressed in daily motion (°/d) in a tabular form, were introduced for the first time into the Chinese calendrical system (line 4). In the following many centuries this tabular demonstration of the Moon's daily variable velocities consistently formed one of the central constituents of all Chinese calendars, as still found with similar values in the Shou Shih calendar (1281–1644; cf. Yuan Shih, ch. 54), well-known for its general accuracy.

As shown above, already in a very early stage the Moon's accurate mean velocity was rendered by the luni-solar relations such as the Metonic cycle, and the year length (1-b, line 13). There should follow, sooner or later, the inevitable perception that the Moon's motion was variable and that the points of maximum and minimum velocity moved around the heavens (Tab. A, 2-, 3-c, -d), leading to a new astronomical concept, the anomalistic month. The Chinese discovered the recurrence of the same lunar velocity every 9 years at the same celestial position which is supposed to have been easily observable (Tab. A, 3-c, -d).

Applying this to the Saros period which would then contain two eastward rotations of the lunar orbit, we obtain

\[ 223 \text{ syn. mon.} \approx 18 \text{ years} \approx (223 + 18) \text{ sid. month} \approx (223 + 18 - 2) \text{ anom. mon.}, \]

leading to the period relation

\[ 223 \text{ syn. mon.} \approx 239 \text{ anom. mon.}. \]

A more precise computation would yield us the number of sidereal rotations of the lunar orbit in 18 years or in one Saros period to be ca. 2.033 \(\approx 2;2\) instead of 2 above. By this or an analogous argument, the Babylonians might have been led to those two crucial period relations, 6247 syn. mon. = 6695 anom. mon. (System A) and 251 syn. mon. = 269 anom. mon. (System B). This conjecture seems promising because soon after their discovery of the 9-year sidereal rotation of the lunar orbit, the so-called “Nine Roads of the Moon” (Tab. A, 3-c, -d), the Chinese came to deduce a more accurate value, the rotation in ca. 8.9 years (Yabuuti, 77, Cullen, 28), as found in the Chhien-Hsiang calendar (Tab. A, line 4), obviously by new observations. For such an improvement the observation of a set of eclipses one, two or three Saroi apart would have been most beneficial, in particular in the case of the Babylonians' discovery of their period relations just mentioned.

Once the period relations between synodic and anomalistic months were established, the Babylonians now inevitably came to introduce the variable lunar velocity into the lunar theory as the most essential constituent which is the origin of the lunar anomalistic motion, causing the length variation of the synodic months. This was done strictly according to their period relations. Crucial for the Babylonian astronomical theory was—not