Abstract Uncertainties are a central element in structural analysis and design. But even today they are frequently dealt with in an intuitive or qualitative way only. However, as already suggested 80 years ago, these uncertainties may be quantified by statistical and stochastic procedures. This contribution attempts to shed light on some of the recent advances in the now established field of stochastic structural mechanics and also solicit ideas on possible future developments.

1 Introduction

The realistic modeling of structures and expected loading conditions as well as the mechanisms for their possible deterioration with time are undoubtedly one of the major goals of structural and engineering mechanics respectively. It has been recognized that this should also include the quantitative consideration of the statistical uncertainties of the models and the parameters involved [56]. There is also a general agreement that probabilistic methods should be strongly rooted in the basic theories of structural engineering and engineering mechanics and hence represent the natural next step in the development of these fields.

It is well known that modern methods leading to a quantification of uncertainties of stochastic systems require computational procedures. The development of these procedures goes along with computational methods in current traditional (deterministic) analysis for the solution of problems required by engineering practice, where computational procedures certainly dominate. Hence, their further development within computational stochastic structural analysis is an important requirement for the dissemination of stochastic concepts into engineering practice. Naturally, procedures to deal with stochastic systems are computationally considerably more involved than their deterministic counterparts, because the parameter set assumes a (finite or infinite) number of values in contrast to a single point in the parameter space. Hence, in order to be competitive and tractable in practical applications, the computational efficiency of procedures utilized is a crucial issue. The significance of this should not be underestimated. Improvements to efficiency can be attributed to two main factors: improved hardware in terms of ever faster computers and improved software, which means improving the efficiency of computational algorithms, which also includes utilizing parallel processing and computer farming. To achieve a continuous increase in efficiency by software development, computational procedures for stochastic analysis should follow a similar path as that followed in the 1970s and 1980s during the development of the deterministic finite-element (FE) approach. One important aspect in this fast development was a focus on numerical methods adjusted to the strengths and weaknesses of numerical computational algorithms. In other words, traditional approaches to structural analysis developed before the computer age have been dropped, redesigned and adjusted to meet the new requirements posed by the computational facilities.
Two main streams of computational procedures in stochastic structural analysis can be observed. The first of this main class is the generation of sample functions by Monte Carlo simulation (MCS). These procedures might be categorized further according to their purpose:

- **Realizations of prescribed statistical information**: Samples must be compatible with prescribed stochastic information such as spectral density, correlation, distribution, etc. Applications are: (1) unconditional simulation of stochastic processes, fields and waves, and (2) conditional simulation compatible with observations and a priori statistical information.

- **Assessment of the stochastic response** for a mathematical model with prescribed statistics (random loading/system parameters) of the parameters. Applications are: (1) representative samples for the estimation of the overall distribution. Indiscriminate (blind) generation of samples. Numerical integration of stochastic differential equations (SDEs); (2) representative samples for reliability assessments by generating adverse rare events with positive probability, i.e. by: (a) variance-reduction techniques controlling the realizations of random variables (RVs), and (b) controlling the evolution in time of sampling functions.

The other main class provides *numerical solutions to analytical procedures*. Grouping again according to the respective purpose the following classification can be made:


In the following, some of the outlined topics will be addressed, stressing new developments. These topics are described within the next six subject areas, each focusing on a different issue, i.e. representation of stochastic processes and fields, structural response, stochastic FE methods and parallel processing, structural reliability and optimization, and stochastic dynamics. In this context it should be mentioned that aside from the Massachusetts Institute of Technology (MIT) conference series the US national congress on computational mechanics (USNCCM), the European conferences on computational mechanics (ECCM) and the world congress on computational mechanics (WCCM) include a larger number of sessions addressing computational stochastic issues.

### 2 Representation of stochastic processes

Many quantities involving random fluctuations in time and space might be adequately described by stochastic processes, fields and waves. Typical examples of engineering interest are earthquake ground motion, sea waves, wind turbulence, road roughness, imperfection of shells, fluctuating properties in random media, etc. For these applications, probabilistic characteristics of the process are known from various measurements and investigations in the past. In structural engineering, the available probabilistic characteristics of random quantities affecting the loading or the mechanical system can often not be utilized directly to account for the randomness of the structural response due to its complexity. For example, in the common case of strong earthquake motion, the structural response will be in general nonlinear and it might be too difficult to compute the probabilistic characteristics of the response by means other than MCS. For the purpose of MCS, sample functions of the involved stochastic process must be generated. These sample functions should represent accurately the characteristics of the underlying stochastic process or fields and might be stationary or nonstationary, homogeneous or nonhomogeneous, one-dimensional or multi-dimensional, univariate or multivariate, Gaussian or non-Gaussian, depending very much on the requirements of accuracy of realistic representation of the physical behavior and on the available statistical data.

The main requirement on the sample function is its accurate representation of the available stochastic information of the process. The associated mathematical model can be selected in any convenient manner as long it reproduces the required stochastic properties. Therefore, quite different representations have been developed and might be utilized for this purpose. The most common representations are, e.g., the autoregressive moving average (ARMA) and autoregressive (AR) models, filtered white noise (SDE), shot noise and filtered Poisson white noise, covariance decomposition, Karhunen–Loève and polynomial chaos expansion, spectral representation, and the wavelets representation.

Among the various methods listed, the spectral representation methods appear to be the most widely used (see, e.g., [71,86]). According to this procedure, samples with specified power spectral density information are generated. For the stationary or homogeneous case the fast Fourier transform (FFT) technique is utilized for a dramatic improvement of computational efficiency (see, e.g., [104,105]). Advances in this field provide