Viscoplasticity of elastomeric materials:
experimental facts and constitutive modelling

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Summary A characteristic of filled elastomers is their ability to undergo very large deformations without damaging their internal structure. The material behaviour is mainly elastic, however, elastomers show hysteresis effects leading to damping properties, which are quite important as regards their applications in various fields of mechanical engineering.

A series of experiments (tension, torsion and combinations of both) was carried out on cylindrical bars made of a carbon-black filled rubber mixture. In addition to a pronounced nonlinear rate-dependence, relaxation and viscosity properties are observed as being influenced by the process histories.

The behaviour of elastomeric materials is modelled on the basis of a free energy function and evolution equations for additional internal variables. Incorporating or disregarding the very small rate-independent hysteresis, the constitutive modelling may be classified under viscoelasticity or viscoelasticity. The constitutive equations are formulated for isothermal processes in a thermodynamically consistent manner. Particular attention is focused on nonlinear rate-dependence as well as on process-dependent relaxation properties. Numerical simulations on the basis of identified material parameters show that the proposed constitutive model is able to represent the main elastic and inelastic phenomena.

Key words Elastomer, rubber, viscoplasticity, viscoelasticity

1 Introduction

Elastomeric or rubberlike materials are applied in various branches of mechanical engineering. The elasticity and damping properties of components made of elastomers influence the structural behaviour of machines and vehicles. A rational modelling and quantitative calculation of the structural behaviour requires physically founded constitutive equations, representing the thermomechanical material behaviour.

Elastomers are able to undergo very large deformations without fracture or other adverse effects on their microstructural integrity. Therefore, it is worthwhile to study the thermomechanical properties of these materials as a subject of the nonlinear continuum mechanics.

Ever since the papers of Mooney, [19] and Rivlin and Saunders, [26], the nonlinear elasticity of elastomeric materials has been the subject of investigation by several authors, [2], [23], [10, 11] and [9]. A compilation of the most frequently used models of isotropic nonlinear elasticity is found in [4, pp. 103–111]. In more recent investigations, the interest was focused on the inelastic, i.e. viscoelastic or viscoplastic behaviour, [17, 12, 9, 6, 24, 25, 18, 1, 29, 30]; see also [13–16] and the literature cited therein.

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The present paper is a report on experimental results which motivate the formulation of a constitutive model of viscoplasticity or viscoelasticity. The type of the constitutive model depends on the size of the equilibrium hysteresis. If the effects of equilibrium hysteresis are recognised to be large enough, the theory of viscoplasticity applies. If, on the other hand, the equilibrium hysteresis appears to be negligible, a viscoelasticity theory is sufficient.

In order to describe the physical foundation of the constitutive model without tensor formalism, the general structure of the constitutive model will first be explained for the uniaxial case. After this, the three-dimensional formulation of the model is straight-forward, if the concept of dual variables, which was proposed in [8], is applied. In the present case, the experiments suggest that the equilibrium hysteresis is relatively small. The numerical simulations demonstrate the possibility of representing the main elastic and inelastic phenomena.

2
Experimental observations
Our experiments were carried out on cylindrical bars, pasted into metallic pots as illustrated in Fig. 1. The specimens were clamped into a device to perform translational and rotational motions. The material was a filled rubber, used in the tyre industry, with a hardness of 74° Shore A. The specimens were provided by CONTINENTAL A.G., Hannover.

A part of the specimen, a circular cylinder of initial length \( L_0 \) undergoes a deformation, which is represented by means of the transformation

\[
(R, \Phi, Z) \mapsto r = \frac{R}{\sqrt[3]{\lambda(t)}}, \quad \varphi = \Phi + D(t)Z, \quad z = \lambda(t)Z.
\]  

(1)

Here, \( R, \Phi, Z \) and \( r, \varphi, z \) denote cylindrical coordinates in the reference and current configuration, respectively. The dimensionless current length

\[
\lambda(t) = \frac{L(t)}{L_0}
\]  

(2)

is the stretch, and the relative rotation \( z \) of the cross sections divided by the initial length \( L_0 \) is the twist

\[
D(t) = \frac{z(t)}{L_0}.
\]  

(3)

The component representation of the deformation gradient (physical components) is calculated from (1) to be

\[
F(x, t) = \begin{pmatrix}
\frac{1}{\lambda(t)} & 0 & 0 \\
0 & 1 & Dr \\
0 & 0 & \lambda
\end{pmatrix}.
\]  

(4)

Fig. 1. Test specimen (filled rubber, 74° Shore A)