Finite element computation of torsional plastic waves in a thin-walled tube

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Summary A finite element technique is presented for the analysis of one-dimensional torsional plastic waves in a thin-walled tube. Three different nonlinear constitutive relations deduced from elementary mechanical models are used to describe the shear stress–strain characteristics of the tube material at high rates of strain. The resulting incremental equations of torsional motion for the tube are solved by applying a direct numerical integration technique in conjunction with the constitutive relations. The finite element solutions for torsional plastic waves in a long copper tube subjected to an imposed angular velocity at one end are given, and a comparison with available experimental results to assess the accuracy of the constitutive relations considered is conducted. It is demonstrated that the strain-rate dependent solutions show a better agreement with the experimental results than the strain-rate independent solutions. The limitations of the constitutive equations are discussed, and some modifications are suggested.

Keywords Finite element technique, Torsional plastic wave, Thin-walled tube, Nonlinear constitutive equation

1 Introduction

The problem of uniaxial elastic–plastic wave propagation in long rods has been an important subject in dynamic plasticity [1] over many decades. More specifically, many experiments on slender bars have been conducted using a longitudinal impact to determine the constitutive equations for materials. It has, however, been well recognized that the stress distributions at the impact end of the bar tend to be three-dimensional due to the radial inertia and transverse shear stresses. Hence, accurate stress–strain relations at strain rates of over $10^4$ s$^{-1}$ cannot be obtained from the longitudinal impact test. In order to overcome the complications encountered in the axial impact, one approach would be to use torsional impact. Consequently, torsional impact testing has been utilized by many investigators. For example, a torsional impact machine was developed, [2], to study the strain-rate effects of pure copper with two different experimental methods. Torsional impact tests were performed using similar experimental techniques on pure copper, [3], and iron, [4]. The strain-rate sensitivity of pure iron was examined in [5], using a new torsional testing machine. Subsequently, the dynamic shear stress–strain relations for several metals were determined with the torsional split Hopkinson bar, see e.g. [6–19]. Torsional plastic waves in a copper tube were analyzed based on three different nonlinear constitutive equations by the method of characteristics [20]. The numerical solutions to elastoplastic shear/compressive waves in a thin-walled cylinder were obtained in [21] with the ADINA computer programs. Recently, a numerical analysis of two-dimensional
torsional waves in an elastic/viscoplastic cylinder was carried out in [22] using the method of bi-characteristics.

The present paper is concerned with a finite element analysis of one-dimensional torsional plastic waves in a thin-walled tube of cold-worked copper. Three different nonlinear constitutive equations deduced from elementary mechanical models are used to represent the shear stress–strain behavior of copper at high rates of strain. The incremental equations of torsional motion for the tube are derived from the principle of virtual work and the d’Alembert’s principle. The resulting equations are solved by applying a direct numerical integration technique combined with the constitutive relations. The numerical results for torsional plastic waves in a tube subjected to an imposed angular velocity at one end are compared with the available experimental data to evaluate the accuracy of the constitutive relations considered.

2
Constitutive relations

2.1
Mechanical models

We consider three kinds of elementary mechanical models to deduce the nonlinear constitutive relations used in the analysis of torsional plastic waves. To simplify the models, the discussion here is restricted to a simple shear stress–strain ($\tau$–$\gamma$) relationship within the framework of the small deformation theory. First, we consider a mechanical model for the elasto-plastic material as shown in Fig. 1. The total strain increment $d\gamma$ in the model consists of an elastic and a plastic strain increment as

$$d\gamma = d\gamma_e + d\gamma_p,$$

which can be rewritten as

$$d\gamma = \left(\frac{1}{G} + \frac{1}{H'_p}\right) d\tau,$$

where $G$ is the shear modulus corresponding to the linear spring, and $H'_p$ is the strain-hardening modulus corresponding to the slope of the static strain-hardening function $g(\gamma_p)$

$$H'_p = \frac{dg(\gamma_p)}{d\gamma_p}.$$

Equation (2) can be regarded as a rate-independent (or finite) constitutive equation as proposed in [23] in incremental form.

Next, we consider two kinds of strain-rate dependent models. A mechanical model for the elasto-viscoplastic material is depicted in Fig. 2, where a nonlinear dashpot (or viscous element) is placed in parallel with plastic (or nonlinear friction) element. The total strain rate $\dot{\gamma}$ in this model is expressed as the sum of the elastic and viscoplastic parts as

$$\dot{\gamma} = \dot{\gamma}_e + \dot{\gamma}_{vp},$$

Fig. 1. Mechanical model for elasto-plastic material (two-element model)

Fig. 2. Mechanical model for elasto-viscoplastic material (three-element model)