Development of plasticity and damage in vibrating structural elements performing guided rigid-body motions

J. Gerstmayr, H. J. Holl, H. Ischik

Summary A numerical algorithm for studying the development of plastic and damaged zones in a vibrating structural element with a large, guided rigid-body motion is presented. Beam-type elements vibrating in the small-strain regime are considered. A machine element performing rotatory motions, similar to an element of a slider-crank mechanism, is treated as a benchmark problem. Microstructural changes in the deforming material are described by the mesolevel variables of plastic strain and damage, which are consistently included into a macroscopic analysis of the overall beam motion. The method is based on an eigenstrain formulation, considering plastic strain and damage to contribute to an eigenstrain loading of a linear elastic background structure. Rigid-body coordinates are incorporated into this beam-type structural formulation, and an implicit numerical scheme is presented for iterative computation of the eigenstrains from the mesolevel constitutive behavior. Owing to the eigenstrain formulation, any of the existing constitutive models with internal variables could in principle be implemented. Linear elastic/perfectly plastic behavior is exemplarily treated in a numerical study, where plastic strain is connected to the Kachanov damage parameter by a simple damage law. Inelastic effects like plastic shakedown and damage-induced low-cycle rupture are shown to occur in the exemplary problems.

Key words Beam, damage, damage-induced rupture, eigenstrain, elasto-plastic vibration, plastic shakedown, rigid-body motion

1 Introduction

Material physics deals with phenomena such as elasticity, which takes place at the level of atoms, with plasticity, which is due to the development of slips at the molecular and crystal level, or damage, which means the development of cavities due to decohesion and debonding. At the mesolevel of continuum mechanics, these microstructural effects and changes in deforming materials are mathematically described by various deformation measures such as elastic and plastic strain, and by internal variables like damage parameters. The influence of these mesoscale deformation variables on the overall motion of engineering structures is considered at the macroscale of structural mechanics. This cascade of theories, formulated at various levels, even nowadays requires bridging gaps between the disciplines involved. From the point of structural mechanics, the goal is to give way the cited mesoscale variables in the macroscopical formulations, but to retain and extend the essential assumptions of structural

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Dedicated to Professor F. G. Kollmann on the occasion of his 65th birthday.

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theories, in order to reflect properly the microstructural changes. As an important example for such a strategy, we refer to the theories for elasto-viscoplastic shells with finite deformations developed by Kollmann et al., see e.g. [1, 2]. As stated in Ref. [1], the aim of these formulations has been the implementability of the existing constitutive models with internal variables in respective shell theories.

The present paper is written in the light of the latter statement. We restrict ourselves to small deformations and to beam-type structural elements exhibiting plasticity and damage. However, we study vibrations produced by large, guided rigid-body motions. Thus, we extend a series of papers presented by our group on elasto-plastic vibrations of structural elements about equilibrium positions, see [3] and [4] for reference. In the formulations and in the papers cited in Refs. [3] and [4], a linear elastic background structure has been introduced at the macroscale, consistently reflecting the fact that microstructural changes are developing in the virgin material. At the macroscale of a vibrating structure, the micromechanics-based variables of both plasticity and damage contribute then to an eigenstrain loading of the linear elastic background structure. The name “eigenstrain” has been introduced by Mura [5], for an incompatible, inelastic part of strain. Such a loading is similar in action to a series of thermal shocks superimposed upon the actual loading. This modelling then enables the use of well-established methods of structural mechanics, also in the presence of advanced constitutive equations. More recently, the method has been extended, e.g. to the case of fluid-structure interaction and to the second-order theory of structures [6, 7].

Our present contribution is motivated by problems of machine dynamics. The aim is to present a computational strategy for the detailed study of plastic shakedown and low-cycle fatigue of machine elements in the time-domain. As a benchmark problem, a large, guided rotatory motion of a hinged-hinged one-span beam is studied in detail. This structure may be imagined as some element of a slider-crank-type mechanism. In Sec. 2 of the present paper, the initial-boundary value problem for flexural vibrations of such a structural element is presented, where the case of a loading by a distribution of eigenstrain is studied in some detail. Since eigenstrains are to be defined locally in each point of the beam, their influence upon the deformations at the level of the beam theory has to be treated with care. Emphasis is also laid on the incorporation of the rotatory rigid-body coordinate into the formulation. Using an expansion into linear eigenmodes, the problem subsequently is approximated by a system of ordinary differential equations in time for the beam deflection. Owing to the underlying structural beam theory, the dimension of the system is much smaller than it had to be expected for a formulation stemming from the three-dimensional theory of continuum mechanics. The system is, moreover, time-variant due to the presence of the rotatory rigid-body motion. A direct solution could be achieved if the eigenstrains would be known in advance. In principle, due to the eigenstrain formulation, any of the existing constitutive models with internal variables is implementable. In the case of vanishing eigenstrains, the formulation corresponds to a purely elastic structure. Convergence of the series expansion is improved by splitting the quasi-static part, which can be calculated from the rotatory rigid-body motion in closed form. The rigid-body motion is assumed to be prescribed without coupling to the beam vibrations. Results for this latter problem, which is currently under investigation, will be reported elsewhere.

In Sec. 3, emphasis is laid on the relation between the eigenstrain loading of the macroscopic beam and the mesoscale deformation variables of plastic strain and damage. The evolution of these variables is first formulated as a function of the total strain of the beam. In turn, the total strains could be evaluated from the deformation of the beam if the solution of the above set of ordinary differential equations, and thus, the eigenstrains, would be known in advance. The eigenstrain loading is subsequently formulated as a function of plastic strain and damage. In order to fix ideas, these relations are exemplarily derived for the case of a linear elastic/perfectly plastic material. Elasto-plastic behavior is related to the Kachanov one-dimensional surface damage parameter by means of the strain equivalence principle introduced by Lemaitre [8]. In the present paper, a simple damage theory, motivated by a formulation of Frantziakis and Desai [9], is used to relate the damage parameter to the plastic strain. Any constitutive modelling which provides relations between the essential mesoscale deformation variables and the characteristic deformation measures of a structural beam theory, could be used instead. Relations presented in Sec. 3 are already suitably formulated for a numerical time-stepping procedure.

Since eigenstrains depend nonlinearly on the beam deformation via the formulations of Sec. 3, a numerical time-integration procedure for the set of ordinary differential equations of Sec. 2 has to be used. A convenient formulation, presented in Sec. 4, is based on the implicit