The effect of mathematical modeling on critical velocity

Abstract The purpose of this investigation was to examine the effects of mathematical modeling on critical velocity (CV) estimates and the oxygen consumption ($VO_2$), heart rate (HR), and plasma lactate values that corresponded to the five CV estimates. Ten male subjects performed a maximal, incremental treadmill test to determine maximal $VO_2$, and four randomly ordered treadmill runs for the estimation of CV. Two linear, two nonlinear, and one exponential mathematical models were used to estimate CV. Regression analyses were used to determine the $VO_2$, HR, and plasma lactate values that corresponded to the five CV estimates from the relationships for $VO_2$, HR, and plasma lactate versus running velocity from the maximal, incremental test. The nonlinear, three-component model (Nonlinear-3) resulted in a mean CV that was significantly ($P < 0.05$) less than the mean values derived from the other four models, and was the lowest CV estimate for each subject. The percent of maximal $VO_2$, HR, and plasma lactate values that corresponded to the Nonlinear-3 model were 89%, 93%, and 63%, respectively. These findings indicate that CV estimates differ by as much as 20% depending upon the model used to determine the characteristics of the velocity/time relationship. Future studies are needed to determine which model provides the most valid estimate of the demarcation point between heavy and severe exercise.

Keywords Cycle ergometry · Heart rate · Oxygen consumption · Lactate · Mathematical modeling

Introduction

Critical velocity (CV) is the treadmill analog of critical power ($W_{\text{crit}}$) for synergistic muscle groups and cycle ergometry (Monod and Scherrer 1965; Moritani et al. 1981). The CV test involves a series of exhaustive runs at different velocities, from which the hyperbolic velocity/time relationship is determined (Fig. 1). Theoretically, the CV test provides estimates of two distinct parameters (Hill and Ferguson 1999; Housh et al. 1991, 1992; Hughson et al. 1984; Pepper et al. 1992): (1) the maximal running velocity that can be maintained for an extended period of time from aerobic energy reconstitution, called the CV, and (2) the distance that can be run using only stored energy sources in the muscle, called anaerobic running capacity (ARC).

Recent studies (Bull et al. 2000; Gaesser et al. 1995; Morton 1996) have demonstrated differences in $W_{\text{crit}}$ estimates depending upon which of five mathematical models were used to analyze the power/time relationship. Gaesser et al. (1995) and Bull et al. (2000) examined two linear models, two nonlinear models and an exponential model, and found that the exponential model resulted in the highest $W_{\text{crit}}$ estimates, and the nonlinear, three-parameter model the lowest.

The mean $W_{\text{crit}}$ estimates from these models differed by approximately 18–24% (Bull et al. 2000; Gaesser et al. 1995). Differences in exercise intensity of this magnitude can have substantial effects on performance factors such as time to exhaustion, as well as the physiological demands of continuous workouts. No studies, however, have compared CV estimates or quantified the physiological demands associated with CV estimates from those five mathematical models. Differences between models may affect the validity of CV as a measure of aerobic function (Hill and Ferguson 1999), as well as the interpretation of CV as the demarcation point between heavy and severe exercise (Gaesser and Poole 1996; Hill and Ferguson 1999; Hill and Smith 1999; Poole et al. 1988, 1990). Therefore, the purpose of this investigation was to
Fig. 1 Regression analysis and curve fits for each of the five mathematical models used to estimate critical velocity (CV) for one representative subject. The models are described in the text. (*ARC* Anaerobic running capacity, \( v_{\text{max}} \) maximum velocity)

examine the effects of mathematical modeling on CV estimates and on the oxygen consumption (\( VO_2 \)), heart rate (HR), and plasma lactate values that corresponded to the five CV estimates.

**Methods**

**Subjects**

Ten male subjects [mean (SD) age = 22 (2) years, height = 179 (4) cm, body mass = 77 (8) kg] who exercised regularly, but were not highly trained (running < 25 km\-week\(^{-1}\)) volunteered for this study. All procedures were approved by the University Institutional Review Board for Human Subjects, and all subjects gave their written informed consent to participate, prior to testing.

**Maximal test**

The subjects performed a continuous incremental treadmill test for the determination of maximal \( VO_2 \) (\( VO_{2\text{max}} \)). The treadmill test began at 6.4 km\-h\(^{-1}\) and thereafter increased by 1.6 km\-h\(^{-1}\) every 3 min up to 14.5 km\-h\(^{-1}\). The intensity was then increased by raising the treadmill grade by 2% every 3 min (with the velocity remaining constant at 14.5 km\-h\(^{-1}\)) until voluntary exhaustion. Throughout the test, the subjects breathed through a Hans Rudolph valve, with the gas volume and concentrations analyzed using a calibrated Horizon Metabolic Measurement Cart (Sensor Medics, Anaheim, Calif., USA). The \( VO_{2\text{max}} \) was defined as the highest measured \( VO_2 \) value given a plateau of \( VO_2 \) with increased work intensity and/or a respiratory quotient > 1.15 (McMiken and Daniels 1976; Williams et al. 1986). The subjects' HR values were monitored throughout the test using a UNIQ CIC Heartwatch system (Leger and Thi- vierge 1988). In addition, blood samples were drawn by a trained phlebotomist, from an indwelling catheter placed in the antecubital vein, during the final 30 s of each treadmill stage. The samples were analyzed for plasma lactate levels using a YSI Model 23L lactate analyzer (Yellow Springs Instrument, Yellow Springs, Ohio, USA).

**CV test**

For the CV test, the subjects completed four randomly ordered treadmill runs to exhaustion on separate days at velocities ranging from 14.5 to 19.3 km\-h\(^{-1}\). The velocities were selected based on the