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Comment on “Volume of magma accumulation or withdrawal estimated from surface uplift or subsidence, with application to the 1960 collapse of Kīlauea volcano” by P. T. Delaney and D. F. McTigue

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Abstract In volcanoes that store a significant quantity of magma within a subsurface summit reservoir, such as Kīlauea, bulk compression of stored magma is an important mode of deformation. Accumulation of magma is also accompanied by crustal deformation, usually manifested at the surface as uplift. These two modes of deformation – bulk compression of resident magma and deformation of the volcanic edifice – act in concert to accommodate the volume of newly added magma. During deflation, the processes reverse and reservoir magma undergoes bulk decompression, the chamber contracts, and the ground surface subsides. Because magma compression plays a role in creating subsurface volume to accommodate magma, magma budget estimates that are derived from surface uplift observations without consideration of magma compression will underestimate actual magma volume changes.

Key words Mogi · Dislocations · Deformation · Magma compressibility · Kīlauea · Gravity variations

Introduction

Repeated geodetic measurements at volcanoes are most commonly interpreted in terms of internal processes. An important geodetic quantity is the volume of ground surface uplift, the integral of vertical displacement over the deformed volcanic edifice, \( \Delta V_{\text{edifice}} \). Confusion exists regarding how to relate this uplift volume to the change in content of an underlying magma chamber, but the paper by Delaney and McTigue (1994) gives valuable guidance. The purpose of this comment is to point out the additional importance of magma compression, volatile exsolution, and gas compression in relating uplift volume to changes in the mass of magma within a reservoir. Magma compression is of such significance that it needs to be taken into account under most circumstances involving estimates of mass change or comparisons of magma storage with erupted volumes.

Delaney and McTigue (1994) examine models for magma-chamber inflation and associated ground-surface uplift. Using solutions for constant-displacement elastic dislocations, they show that the ratio of the volumes of ground-surface uplift and chamber inflation depend greatly upon chamber geometry. For a spherical chamber (Mogi 1958), uplift volume \( \Delta V_{\text{edifice}} \) is \( \frac{3}{2} \) the change in chamber volume \( \Delta V_{\text{ch}} \) (referred to as \( \Delta V_{\text{injection}} \) by Delaney and McTigue 1994) if, as is generally done, Poisson’s ratio of the host rock is taken as 0.25. The result demonstrates that host rocks around a spherical chamber undergo a bulk dilation proportional to inflation. In contrast, inflation of sill-like and dike-like chambers produces volumes of uplift equal to and less than, respectively, the change in source chamber volume \( \Delta V_{\text{ch}} \). The lesson is that uplift volume, which can be estimated from data independently of any physical models of the underlying magma reservoir, does not correspond simply to the volume change of a source at depth.

Yet, just as host rocks are dilated or compressed by changing magma-reservoir pressures, so is the magma stored within the reservoir (Eggers 1983; Johnson 1992, 1995; Sanderson 1982; Sanderson et al. 1983). Because compression of magma resident within the chamber can create additional space to accommodate injected magma without enlargement of the chamber, the...
change in chamber volume that causes inflation requires a greater mass of magma than is implied by the treatment of Delaney and McTigue (1994).

Analysis of volume change associated with volcanic activity involves a confusing array of terms and definitions. It is important to note that chamber inflation \( \Delta V_{\text{ch}} \) is the volume change of the source cavity itself. We suggest the nomenclature \( \Delta V_{\text{ch}} \) of Sigmundsson et al. (1992) as a replacement for the term "injection volume" \( \Delta V_{\text{injection}} \) of Delaney and McTigue (1994) when referring to volumetric expansion of the source chamber. The volume of ground-surface uplift \( \Delta V_{\text{edifice}} \) (or \( \Delta V_{\text{uplift}} \) of Delaney and McTigue 1994) is composed of the combination of volume displaced by chamber expansion \( \Delta V_{\text{ch}} \) plus deformation-induced crustal dilation. Delaney and McTigue (1994) examine the factors influencing \( \Delta V_{\text{edifice}} \) in detail.

Now we turn to terms that address magma volumes. The volume of magma that enters the chamber in the case of inflation, or exits the chamber in the case of deflation, is termed \( \Delta V_{\text{magma}} \). Variation in chamber pressure that accompanies intrusion and extrusion produces volumetric compression or decompression of stored magma. The cumulative effect of very slight compression of a relatively large amount of magma contained within the reservoir may amount to a significant net volume change of stored magma, \( \Delta V_{\text{compression}} \). A parcel of magma \( \Delta V_{\text{magma}} \) intruded into a chamber is accommodated by a combination of expansion of the chamber and compression of stored magma. The rule is that 

\[
\Delta V_{\text{magma}} = \Delta V_{\text{ch}} + \Delta V_{\text{compression}}
\]

Finally, outside the scope of this comment are magma volume changes, due largely to gas expansion, which may occur during magma transport to the low-pressure environment near the Earth's surface.

An expression for surface uplift due to magma injection into a spherical chamber can be derived by relating equations for ground-surface uplift and magma compression in response to an increment in pressure in a spherical chamber. This has been done by Tryggvason (1981), Johnson (1987), and Sigmundsson et al. (1992).

A more general equation, taking into account the effects of volatiles in the magma, was derived by Johnson (1992) for a situation in which the amount of injected magma is small relative to the amount of magma originally in the chamber. For a spherical chamber, the ratio between the volume of uplift \( \Delta V_{\text{edifice}} \) and the volume of the parcel of magma intruded into (or withdrawn from) the reservoir \( \Delta V_{\text{magma}} \) is given by

\[
\frac{\Delta V_{\text{edifice}}}{\Delta V_{\text{magma}}} = \frac{2(1 - \nu)}{1 + \frac{4\mu}{3K^*}}
\]

where \( \nu \) and \( \mu \) are Poisson's ratio and the shear modulus of the host rock, respectively, and \( K^* \) is an effective bulk modulus of the magma stored within the reservoir and is defined as

\[
K^* = K \quad \text{for} \quad N > N_s
\]

(modified from Johnson 1992). Equation (2b) introduces the specific effect of volatiles on \( K \), the gas-free melt bulk modulus. In magma chambers with depths of ~3 km, CO\(_2\) is the volatile most likely to be present in reservoir magma as a gas phase (Gerlach and Graeber 1985) and thus to affect the value of \( K^* \). The total weight fraction of CO\(_2\) dissolved and exsolved in the magma is given as \( N \), and \( N_s \) is the limiting amount of CO\(_2\) that may be dissolved in the melt. In Eq. (2b), \( \rho_m \) is the bulk-magma density, \( R = 8.314 \text{ m}^3 \text{Pa/mol °K} \) is the gas constant, \( T \) is absolute temperature, \( P \) is average pressure (assumed to be lithostatic), and \( \omega \) is molar mass of gas. The molar mass of CO\(_2\) is \( \omega = 0.044 \text{ kg} \).

Equation (2b) assumes that the pressure change is small relative to the total pressure \( P \) and that the pressure change does not cause \( N_s \) to pass \( N \).

Equation (1) shows that the properties determining the relative importance of crustal deformation and magma compression during inflation are the crustal shear modulus \( \mu \) and effective magma bulk modulus \( K^* \), regardless of chamber size and depth. As \( \mu K^* \rightarrow 0 \), the results obtained by Delaney and McTigue (1994) for an incompressible magma are retrieved, the ratio of uplift volume to injection volume being

\[
\frac{\Delta V_{\text{edifice}}}{\Delta V_{\text{magma}}} = 2(1 - \nu) > 1 \quad \text{for all} \quad \nu \quad \text{and} \quad \frac{\Delta V_{\text{edifice}}}{\Delta V_{\text{magma}}} = 1.5 \quad \text{for} \quad \nu = 0.25
\]

Comparing the effect of host-rock expansion and magma compression, this ratio is less than unity, \( \frac{\Delta V_{\text{edifice}}}{\Delta V_{\text{magma}}} < 1 \), if \( \frac{\mu}{K^*} > \frac{3(1 - 2\nu)}{4} \) and if \( \frac{\mu}{K^*} > \frac{3}{8} \) for \( \nu = 0.25 \). In the particular case where \( \frac{\mu}{K^*} = \frac{3}{8} \) and \( \nu = 0.25 \), the net volume change due to crustal dilation is equal in value with opposite sign to the net volume change due to compression of stored chamber magma. Only in this special situation is \( \Delta V_{\text{edifice}} \) identical to \( \Delta V_{\text{magma}} \).

Consider the case of a magma undersaturated with respect to CO\(_2\), \( N < N_s \), so that \( K^* = K \) (Eq. (2a)). Several measurements of magma bulk modulus \( K \) exist for gas-free Kilauea 1921 olivine tholeiite (Fuji and Kushiro 1977; Murase et al. 1977), ranging from 11.5 to 17 GPa from atmospheric pressure to 1.5 GPa. Piezomagnetic observations (Davis et al. 1974) and analysis of dike widths (Rubin and Pollard 1987) suggest the effective shear moduli of Kilauea's edifice is less than the ~25 GPa determined in the laboratory using intact samples (Manghnani and Wollard 1968; Ryan 1987). For the purpose of an example, a low shear modulus of 3 GPa, consistent with Davis et al. (1974) and Rubin and Pollard (1987) and a gas-free magma bulk modulus of 12 Gpa, gives \( \frac{\mu}{K^*} = 0.25 \) and \( \frac{\Delta V_{\text{edifice}}}{\Delta V_{\text{magma}}} = 1.1 \) for \( \nu = 0.25 \). In this example, compression of magma nearly