Fair versus Unrestricted Bin Packing

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Abstract. We consider the on-line Dual Bin Packing problem where we have \( n \) unit size bins and a sequence of items. The goal is to maximize the number of items that are packed in the bins by an on-line algorithm. We investigate unrestricted algorithms that have the power of performing admission control on the items, i.e., rejecting items while there is enough space to pack them, versus fair algorithms that reject an item only when there is not enough space to pack it. We show that by performing admission control on the items, we get better performance compared with the performance achieved on the fair version of the problem. Our main result shows that with an unfair variant of First-Fit, we can pack approximately two-thirds of the items for sequences for which an optimal off-line algorithm can pack all the items. This is in contrast to standard First-Fit where we show an asymptotically tight hardness result: if the number of bins can be chosen arbitrarily large, the fraction of the items packed by First-Fit comes arbitrarily close to five-eighths.

Key Words. On-line algorithms, Competitive analysis, Bin Packing, Dual Bin Packing, Restricted adversaries, Randomization, Admission control.

1. Introduction

The Problem. Bin Packing is one of the most classical problems in combinatorial optimization and in theoretical computer science. In the Classical Bin Packing problem we are given an unlimited number of unit bins and a set of items each with a non-negative size where the goal is to minimize the number of bins used to pack all the items. In the Dual Bin Packing problem we are given a fixed number \( n \) of unit size bins and a set of items each with a non-negative size where the goal is to maximize the number of items packed. The Dual Bin Packing problem has been studied in the off-line setting, starting in [9], and its applicability to processor and storage allocation is discussed in [8]. (For surveys on Classical Bin Packing, see [7] and [10].) In the on-line version of the problem, the items arrive in some sequence and the assignment of an item should be done before the next item arrives.
In this paper we consider the on-line Dual Bin Packing problem. In this problem the algorithm may not be able to pack all items and the question is whether the algorithm is allowed or not allowed to perform admission control. The Fair Bin Packing problem was investigated in [6]. In Fair Bin Packing an algorithm is only allowed to reject an item if it cannot fit in any bin at the time it is given. Note that, for this version of the problem, the off-line algorithm is also required to be fair. In this paper we also consider what happens when the fairness restriction is removed and call the problem Unrestricted Bin Packing.

The Performance Measures. The standard measure for the quality of on-line algorithms is the competitive ratio. For the Bin Packing problem, the competitive ratio of an algorithm $A$ is the worst case ratio, over all possible input sequences, of the number of items packed by $A$ to the number of items packed by an optimal off-line algorithm.

For the Bin Packing problem, as well as for many other on-line problems, the competitive ratio yields very pessimistic results. In particular, for the maximization problems no algorithm can pack a constant fraction of the number of items packed by the optimal algorithm and the competitive ratio must depend on the size of the smallest item (see [1], [2], and [6], for example). Since we are interested in results that hold for arbitrary size items (and get constant competitive ratios similar to most of the results in the Classical Bin Packing problem), we need to restrict the input sequences. Having in mind the Classical Bin Packing problem where all items are required to be packed, a natural assumption is to restrict the input sequences to those which can be completely packed by an optimal off-line algorithm. This enables us to obtain significantly better results. Such sequences are called accommodating sequences, since the off-line algorithm can accommodate the whole sequence.

Note that on accommodating sequences, the competitive ratio of Unrestricted Bin Packing is no worse than the competitive ratio of the fair problem, since the off-line algorithm packs all items and hence is fair. In general, however, the competitive ratio of Unrestricted Bin Packing is not necessarily better than the competitive ratio of the fair problem since the off-line algorithm can also benefit from not being fair. In fact, in many cases, considering unfair algorithms, i.e., performing admission control on the items, is the more challenging problem; see for example the results for throughput routing in [1]–[3]. In particular, with the Unrestricted Bin Packing problem, the competitive ratio of different algorithms can vary over a large range. This is in contrast to on-line algorithms for Fair Bin Packing where all competitive ratios for deterministic algorithms are within a constant factor of each other, both for arbitrary sequences and for accommodating sequences (see [6]).

The competitive ratio and accommodating sequences are defined formally in Section 2.

The Results. The results in this paper are for accommodating sequences for the online Dual Bin Packing problem. The Fair Bin Packing problem is considered in [6] by analyzing the First-Fit algorithm, where each item is packed in the lowest index bin into which it fits and rejected if it does not fit in any bin. It is shown in that paper that First-Fit has a competitive ratio of at least $5/8$ on accommodating sequences, i.e., it packs at least $5/8$ of the items. In this paper we show that the bound is asymptotically tight, i.e.,