Computational \( p \)-element method on the effects of thickness and length on self-weight buckling of thin cylindrical shells via various shell theories

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Abstract This paper is concerned with the development of a global \( p \)-element method for the analysis of self-weight buckling of thin cylindrical shells. Such buckling problems occur when cylindrical shells are subject to high-\( g \) acceleration, for instance the launching of rockets and missiles under high propulsive power. The cylindrical shells may have any combination of free, simply supported and clamped ends. A \( p \)-element computational method has been developed based on various thin shell theories including Donnell, Sanders and Goldenveizer-Novozhilov models. The strain energy for the global element during buckling is formulated and an eigenvalue equation is derived. Unlike the conventional buckling problem where the eigenvalue is directly solved, a pre-determined buckling parameter is fixed at the outset for a geometric-dependent stiffness and a recursive numerical procedure is developed to compute the effect of critical buckling length. The critical buckling length is found to be proportional to thickness to a power of approximately 0.9. The effects of shell thickness and length on buckling parameter are also investigated. Comparison of results from various shell theories indicates solutions of the Sanders and Goldenveizer-Novozhilov shell theories are in excellent agreement while the Donnel shell theory is good for buckling of short cylindrical shells.

Keywords Buckling, Cylindrical shell, Eigenvalue, Self-weight, Thin shell

1 Introduction

The practical importance of circular cylindrical shells in engineering has made their buckling capacity analysis essential in the planning process. For such reasons, the buckling problems under self-weight or body-force are confronted in many applications, especially in aerospace engineering and military high propulsive weapon research, i.e. in rocket and missile launching where high-\( g \) acceleration is encountered. This is particularly critical during re-entry of aerospace structures into the atmosphere because of the high magnitude of deceleration and, more importantly, the generation of excessive heat causes the Young's modulus to decrease significantly and thus the buckling capacity. Although the effect of excessive heat is not in the scope of this study, the reduction in Young's modulus due to heat poses great threat of self-weight buckling. The elastic buckling problem of bars and columns was first solved by Euler (1774) and it continues to be a subject of interest after more than two centuries for Timoshenko and Gere (1961), Bulson (1970), Wang and Ang (1988), Wang et al. (1993). On the other hand, the elastic buckling of cylindrical shells under axial pressure was pioneered much later in the 1900s by Lorenz (1908) and Timoshenko (1910). The research in shell buckling was particularly intensive over the last several decades as documented in the Structural Stability Handbook compiled by C.R.C.J (1971), standard texts on stability of plates and shells, for example Timoshenko and Gere (1961), and technical papers (Sobel 1964; Wang and Liew 1991; Wang et al. 1992; Ohta and Narita 1993, 1994; Wang et al. 1994; Teng 1996; Tian et al. 1999; and Liew and his co-workers, 1992, 1993, 1994, 1996a, b). Buckling of composite cylindrical shells were investigated by Khdeir and Reddy (1989), Reddy and Starnes (1993), and Kassegne and Reddy (1998). Although the elastic buckling of columns and long-deep beams under self-weight has been investigated (Timoshenko and Gere 1961; Ang and Wang 1990; Teng and Yao 2000; Tan 2000), to the best of the authors' knowledge, there has been very little study on buckling of cylindrical shells under their own weight, particularly the knowledge of effect on critical buckling length. This buckling problem has significant engineering implications and may be encountered when the shells are used in a high-\( g \) environment such as the launching of rockets and missiles under a high propulsive power.

It is worth noting that buckling of cylindrical shells due to self-weight has been less elaborated. A recent survey by Teng (1996) included more than 300 publications in recent years but it is silent on self-weight buckling of cylindrical shells. It is believed that the omission of this subject in the review article is due to the lack of references. A comprehensive literature survey reveals only a few relevant publications (Weingarten 1962; Johns 1966; Calladine and Barber 1970; Johns 1973; Mandal and Calladine 2000) published on the subject of buckling of cylindrical shells.
under self-weight/body force. The earliest article on record is that of Weingarten (1962) on the effect of nonuniformity of loading on the buckling characteristics of circular cylinders. Later, Johns published two brief notes (Johns 1966; Johns 1973) on the reasoning of the applicability of classical buckling results to linear buckling analyses for asymmetric axial compressive stress. Calladine and Barber (1970) conducted simple experiments to investigate the self-weight collapse of vertical, thin cylindrical shells with rigid bases and open tops. They concluded that the argument of equating the total weight of shell to the buckling load of a weightless cylindrical shell (Timoshenko and Gere 1961) may be highly unsafe if the mode of buckling involves inward collapse of the open end. The experimental study was revisited by Mandal and Calladine (2000) where simple experiments on self-weight buckling of thin, open-top, fixed-base, small-scale silicone rubber cylindrical shells were conducted.

Owing to the dearth in analysis of cylindrical shells under their own weight, despite its importance in aerospace engineering involving a high-g environment, this paper presents a computational approach of self-weight buckling of cylindrical shells. A global p-element computational method has been developed based on various shell theories including the Donnel, Sanders and Goldenveizer-Novozhilov thin shell models. The strain energy for the global element during buckling is formulated and a governing eigenvalue equation is derived. Unlike the conventional buckling problem where the eigenvalue is directly solved, a pre-determined buckling parameter is fixed at the outset for a geometric-dependent stiffness and a recursive numerical procedure is developed to compute the effect of critical buckling length. The shells treated herein may be subjected to differential combinations of boundary conditions, namely free, simply supported or clamped. The buckling length is found to be proportional to thickness to a power of approximately 0.9. Using this p-element method, the effects of shell thickness and length on buckling parameter are also investigated. Comparison of results from various shell theories indicates solutions of the Sanders and Goldenveizer-Novozhilov shell theories are in excellent agreement while the Donnel shell theory is good for buckling of short cylindrical shells. Numerical results provide a qualitative explanation why the trunk of a tree is comparatively larger at the bottom near the ground.

The computational p-element method presented in this paper has distinct advantages over the classical approaches (Timoshenko and Gere 1961; Bulson 1970; Weingarten 1962; Johns 1966; Calladine and Barber 1970; Johns 1973; Mandal and Calladine 2000). One of the main advantages is that the method presented here is applicable to any combinations of free, simply supported and clamped boundaries at the end whereas the classical method can only solved column and shell buckling with limited boundary conditions, such as cantilever and simply supported shell at both ends (Timoshenko and Gere 1961; Bulson 1970). Another significant conclusion of this paper is that the buckling heights were found to be proportional to \((h/R)^{0.9}\), compared to \((h/R)^{1.0}\) as in the classical theory.