Monolithic and partitioned time integration methods for real-time heterogeneous simulations

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Abstract Real-time (RT) heterogeneous simulations define a class of hybrid numerical–experimental techniques based on dynamic substructuring and capable of simulating the nonlinear response of an emulated mechanical system. With this objective in mind, we present two direct coupling algorithms endowed with subcycling, capable of ensuring the continuity of acceleration between non-overlapping subdomains. In greater detail, firstly we introduce monolithic Rosenbrock L-stable algorithms and, in view of the analysis of complex emulated systems, we recall a recent direct parallel algorithm. Secondly, we propose an improved parallel version of the progenitor algorithm together with its solution procedure. Consequently, in order to reduce drift, we introduce a mass-orthogonal velocity projection characterized by a non-negative energy dissipation. Moreover, both a convergence analysis on a SDoF test problem and simulations on single- and four-DoF systems are presented. Lastly, a novel test rig devised to perform nonlinear substructured RT tests is introduced and a few test results are presented.

Keywords Dynamic · Projection schemes · Algorithms · Numerical methods

1 Introduction

1.1 Background and motivation

In recent years there was the need to analyse, develop and deploy in the market high quality products as quickly as possible, and to develop efficient approaches to the synthesis of such engineering systems. In view of these developments, several tools and projects have been recently developed. Distinguishing features were: (i) the use of partitioned techniques for structure–structure interaction problems [19,39]; (ii) partitioned techniques for fluid–structure interaction problems [25–27,40]; (iii) modular and distributed simulations of multibody systems [1,42]; (iv) real-time (RT) simulations [5]; and (v) hardware-in-the-loop (HIL) and RT heterogeneous simulations based on the principle of dynamic substructuring (DS) [6,41]. For instance, Kassiotis et al. [26] proposed a partitioned solution approach for nonlinear fluid–structure interaction problems. The approach was based on the direct force–motion transfer whilst the problem of enforcing the fluid–structure interface matching was handled by the fixed-point strategy with an adaptive relaxation parameter. A direct proof was made on the stability of the proposed implicit algorithm valid for the fully nonlinear fluid–structure interaction problem. The approach was based on the direct force–motion transfer whilst the problem of enforcing the fluid–structure interface matching was handled by the fixed-point strategy with an adaptive relaxation parameter. A direct proof was made on the stability of the proposed implicit algorithm valid for the fully nonlinear fluid–structure interaction problem. Kassiotis et al. [27] suggested the partitioned approach because of its modularity and the possibility of re-using existing software developed for each subproblem. With regard to the semidiscretization in time, the element-based partitions, that assign different sets of elements to different subdomains of the mesh and create an interface of shared nodes between non-overlapping subdomains, are very popular. In greater detail, the finite element tearing and interconnecting (FETI) method emerged both as one of the most powerful iterative solvers for elliptic problems and a most popular domain decomposition method.
for quasi-static structural mechanics problems [17]; later, the FETI was extended to dynamic problems [18]. Its success derives from the fact that it is based on a dual Schur-type approach, where equilibrium of interface forces is enforced with Lagrange multipliers and from the fact that intersubstructure field continuity generates a small-size symmetric dual problem with unknown Lagrange multipliers. These methods were applied first to the Euler–Lagrange form of the equations of motion. In particular, Gravouil and Combescure [20] conceived a multi-time-step coupling method based on the velocity continuity at the interface, labelled as the GC method, able to couple arbitrary Newmark schemes with different time steps in different subdomains. In this context, they proved that the GC method is unconditionally stable, as long as all individual subdomains satisfy their own stability requirements. Moreover, they showed that for multi-time-step cases, the GC method entails first-order accuracy and energy dissipation at the interface; whereas for the case of a common single time step in all subdomains, the GC method is second-order accurate and energy preserving. The decrease of accuracy for the multi-time-step case, caused by the corresponding computation of the Lagrange multiplier vector, was recently proved by Jia [23]. Many applications were made through this method [14]; and, recently, both a general formalism which enables the coupling of subdomains with heterogeneous time integrators and incompatible time steps was suggested and zero numerical dissipation was ensured at the interface [30,31]. As a result, the global stability of the coupling method is governed by the stability of each time integrator without influence of the interface. Mahjouri and Krenk [29] also proposed a new multi-time step integration method based on different time steps and/or different state-space time integration schemes. Users may control the algorithmic damping introduced in each subdomain by locally choosing the most suitable algorithmic parameters, by means of either the so-called balanced dissipation algorithm or the so-called high-frequency algorithm. These algorithms were extended to include constraints via Lagrange multipliers written in a Schur dual formalism. The change in energy at the interface is identically zero by construction, and thus coupling does not affect the global stability of the problem. The aforementioned methods result in subdomains almost completely independent of each other, that can be separately solved, and coupled together by imposing the continuity of some physical quantities at shared nodes. As a result, these algorithms are amenable to HIL and RT simulations—testing—being able to deal with heterogeneous sub-systems. Nonetheless, staggered solution procedures where considered a drawback in either RT or parallel computations. In order to solve this issue, Pegon and Magonette proposed an interfield parallel solution procedure, which led to a new direct coupling method, the so-called PM method [38]. The favourable convergence properties of this method were thoroughly analysed in Bonelli et al. [3]. The interaction of numerical substructures (NS) and physical substructures (PS) by means of actuators in HIL and RT testing, see the block diagram in Fig. 1, naturally leads to the analysis of coupled-field problems, that are primarily control–structure interaction (CSI) problems [9,37]. In that case, relevant state equations are typically expressed in Hamilton form; moreover, numerical and control requirements impose different time steps for a NS and a PS, respectively. Notably, partitioned algorithms must operate in RT, where the computational time per time interval—i.e. the integration time $\Delta t$ plus the time for display—must be smaller than or equal to the physical time taken by the actual motion of the whole system; and they must separately integrate the long integration process typical of complex NS and the fast process of PS imposed on by the control system of actuators that operates at sampling frequencies of about 2,000 Hz. Within the framework of partitioned Runge–Kutta-based integrators, Nakshatrala et al. [34] proposed an FETI-based coupling method capable of accommodating different time integrators and time steps in diverse subdomains. This was made possible by using a differentiated kinematic constraint and by rendering explicit the calculation of interface Lagrange multipliers. The proposed methods were non A-stable and made use of stabilization techniques that often depended on arbitrary constants. In [35], the authors presented similar algorithms with implicit–explicit coupling, in order to use explicit integrators with fine time steps in stiff mesh regions and implicit integrators with coarse time steps in non-stiff mesh regions. In conjunction with HIL techniques, we recall novel monolithic linearly implicit Rosenbrock-based methods applied to both linear and nonlinear systems [8,10], which require only a single linearization and matrix decomposition per time step [21, p. 102]. In fact, it is worthwhile to recall that in the case of RT computations, the computational time per interval—i.e. the integration time $\Delta t$, plus the time for display—must be smaller than or equal to the physical time taken by the actual motion of an emulated system. Therefore, fully implicit algorithms typically employed in simulations cannot be used. Successively, Jia et al. [24] devised novel element-based partitioned algorithms for staggered and parallel computations able to solve DAEs in Hamilton form. These methods are based on differentiated kinematic constraints and utilize small time steps $\Delta t$ for HIL applications; therefore, first they solve interface Lagrange multipliers and second they advance the solution in all the subdomains, separately.

1.2 Scope

Though the aforementioned direct partitioned algorithm [24] proved to be successful from a numerical standpoint, there