Quantitative simulation of gas-particle two phase plane mixing layer using discrete vortex method


Abstract The gas and particle phase in a two-phase plane mixing layer flow are numerically simulated using the discrete vortex method and a trajectory tracking method. It is shown that the number of vortex elements contained in two semi-infinite discrete vortex sheet and the method of generating control volumes for statistical calculation of the particle phase have important effects on the predicted results of particle phase, especially for quantitative prediction. By adopting different number of vortex elements for two semi-infinite discrete vortex sheet and overlapping the control volumes, predicted results including streamwise velocity, fluctuating velocity and Reynolds shear stress of both phases are obtained and agree well with experimental measurements quantitatively. It shows that the discrete vortex method can achieve the accurate quantitative simulation of two-phase flow.

Introduction

Large coherent vortex structures exist in many types of turbulent flows, especially in mixing layers, jets and wakes, which have been well recognized by many experiments such as Brown and Roshko (1974), Winat and Browand (1974) and Longmire and Eaton (1992). These large coherent vortex structures play an important role in the momentum and energy transportation in turbulent flows. Contrary to the dispersion of particles in isotropic, homogeneous turbulence, the dispersion of particles in mixing layer is dominated by the forming, growth and interaction of large vortex structures. The traditional approach by the Fickian diffusion law cannot model the dispersion of particles properly. The discrete vortex method is grid-free and is capable of characterizing large vortex structures effectively without introducing any turbulent models. It has been used to simulate flows consisting mainly of large vortex structures. These simulations of single-phase flow using discrete vortex method have produced results that agree well with experiments qualitatively and quantitatively (Ghoniem and Ng, 1987; Inoue 1985; Kounoutsakos and Leonard, 1995). However, applications of discrete vortex method on particle dispersion in turbulent flows are only limited to qualitative studies.

Chein and Chung (1987) studied the effect of vortex pairing in a temporally developing mixing layer and their results showed that particle dispersion is strongly dependent on the particle Stokes number. Particles with very small Stokes number dispersed essentially as fluid elements while very heavy particles dispersed less than the fluid. For Stokes numbers in the range of 0.5 to 5, the particles dispersed more than the fluid. Chein and Chung (1988), Wen et al. (1992) as well as Ory and Perkins (1997) extended the work to spatially developing plane mixing layers and studied the influence of large vortex structures and Stokes numbers on particle dispersion in various turbulent flows. Tang et al. (1992) and Chung and Truitt (1988) studied the dispersion of particle in plane wake and axisymmetric jet, respectively. Although these studies have obtained similar conclusions on the dependency of particle dispersion on Stokes number as that of Chein and Chung (1987), they have not been able to obtain quantitative results that agree well with the experiments.

In this paper, gas-particle two-phase plane mixing layer flow is simulated using discrete vortex method and the trajectory tracking method and are shown to be in good agreement with experimental data quantitatively for both phases. This indicates that the discrete vortex method has the ability of simulating two phase flow quantitatively.

Discrete vortex method

In the vortex blob method, the continuous vorticity field is discretize into a finite number of vortex elements with overlapping cores such that

$$\omega(\vec{X}, t) = \sum_{j=1}^{N} \Gamma_j \gamma(|\vec{X} - \vec{X}_j(t)|)$$, \hspace{1cm} (1)

where $N$ is the total number of vortex elements, $\vec{X}_j = (x_j, y_j)$ is the location of the $j$th vortex element, $\Gamma_j$ is its respective circulation, and $\gamma(s)$ is the core Gaussian distributed function describing vorticity within a vortex element as employed by Wang et al. (2000).

The gas-phase velocity induced by all the discrete vortex elements is given by

$$\vec{V}(\vec{X}, t) = \sum_{j=1}^{N} \vec{k} \times \Gamma_j \frac{\vec{X} - \vec{X}_j(t)}{|\vec{X} - \vec{X}_j(t)|^2}F(|\vec{X} - \vec{X}_j(t)|)$$, \hspace{1cm} (2)
where

\[ F(r) = \frac{1}{2\pi} \left[ 1 - \exp \left( -\frac{r^2}{\sigma^2} \right) \right] . \]  

(3)

The viscous diffusion is treated by the diffusion velocity method introduced by Ogami and Akamatsu (1991). For an incompressible, two-dimensional viscous flow, the equation for the transport and diffusion of vorticity \( \omega \) can be written as

\[ \frac{\partial \omega}{\partial t} + (\vec{V} \cdot \nabla) \omega = \nu \nabla^2 \omega . \]  

(4)

Combining the convection term and the diffusion term in the above equation and rewrite it as the following form

\[ \frac{\partial G}{\partial t} + \frac{\partial (uG)}{\partial x} + \frac{\partial (vG)}{\partial y} = 0 , \]  

(5)

then the vorticity transport equation (4) can be written as

\[ \frac{\partial \omega}{\partial t} + \frac{\partial}{\partial x} \left[ \left( u - \frac{\omega}{\omega_c} \cdot \frac{\partial \omega}{\partial x} \right) \omega \right] + \frac{\partial}{\partial y} \left[ \left( v - \frac{\omega}{\omega_c} \cdot \frac{\partial \omega}{\partial y} \right) \omega \right] = 0 . \]  

(6)

Comparing Eqs. (5) and (6), the vorticity function \( \omega \) can be considered as moving with total velocities \( u - \frac{\omega}{\omega_c} \frac{\partial \omega}{\partial x} \) in the x-direction and \( v - \frac{\omega}{\omega_c} \frac{\partial \omega}{\partial y} \) in the y-direction, where \( u \) and \( v \) are the usual convective velocities. Hence, the effect of viscosity is to add a diffusion velocity component \( \vec{V}_d = (u_d, v_d) \) to the motion of each vortex, where \( u_d \) and \( v_d \) are given by

\[ u_d = -\frac{\omega}{\omega_c} \frac{\partial \omega}{\partial x} \quad \text{and} \quad v_d = -\frac{\omega}{\omega_c} \frac{\partial \omega}{\partial y} . \]  

(7)

Therefore, in the diffusion velocity model of the discrete vortex method, the discrete vortex element is transported both by the convection velocity \((u, v)\) and by the diffusion velocity \((u_d, v_d)\), and its circulation is kept invariant along its trajectory according to Eq. (6). Substituting Eq. (1) into Eq. (7), the diffusion velocity \((u_{d\text{r}}, v_{d\text{r}})\) at the coordinates \((x_i, y_i)\) is given as

\[ u_{d\text{r}} = \frac{2v}{\pi \sigma^4 \omega_i} \sum_{j=1}^{N} \Gamma_j (x_i - x_j) \exp \left[ -\frac{(x_j - x_i)^2 + (y_j - y_i)^2}{\sigma^2} \right] \]  

(8)

and

\[ v_{d\text{r}} = \frac{2v}{\pi \sigma^4 \omega_i} \sum_{j=1}^{N} \Gamma_j (y_i - y_j) \exp \left[ -\frac{(x_j - x_i)^2 + (y_j - y_i)^2}{\sigma^2} \right] . \]  

(9)

After obtaining the convective velocity \( \vec{V}_d \) and the diffusion velocity \( \vec{V}_{d\text{r}} \) of the \( i \)th vortex element, its position can be determined by integrating the equation

\[ \frac{d \vec{X}_i(t)}{dt} = \vec{V}_i(t) + \vec{V}_{d\text{r}}(t) . \]  

(10)

Upon integration and using the second-order Runge-Kutta scheme, the following is obtained

\[ \vec{X}_i(t + \Delta t) \approx \vec{X}_i(t) + \left[ \vec{V}_i(\vec{X}_i(t) + h_i) + \vec{V}_{d\text{r}}(\vec{X}_i(t) + h_i) \right] \Delta t \]  

(11)

where \( \Delta t \) is the time step and \( h_i = \frac{1}{2} [\vec{V}_i(\vec{X}_i(t)) + \vec{V}_{d\text{r}}(\vec{X}_i(t))] \Delta t \).

**Trajectory tracking method**

The two-phase flow under investigation is treated as a dilute flow. The assumptions are

- all particles are rigid spheres with identical diameters \( d_p \) and density \( \rho_p \);  
- density of particles is much greater than that of the fluid;  
- particle–particle interactions are negligible;  
- effect of particles on the flow is neglected.

Based on the above assumptions, it is generally accepted that the dominant force on each particle is the drag force in the ambient fluid. Consequently, forces on the particles due to virtual mass, pressure gradient and the Basset force are neglected in this simulation. The equation of motion for a particle is thus

\[ \frac{d \vec{V}_p}{dt} = \frac{f}{(\rho_p d_p^2/18 \mu)} (\vec{V}_f - \vec{V}_p) , \]  

(12)

where \( \vec{V}_f \) is the velocity of a particle, \( \vec{V}_f \) the velocity of fluid at the particle’s position, \( f \) the modification factor for the Stokes drag coefficient and \( \mu \) the fluid dynamic viscosity. The factor \( f \) is represented for particle Reynolds numbers less than 1000 by

\[ f = 1 + 0.15 \text{Re}_p^{2/3} , \]  

(13)

where \( \text{Re}_p \) is defined as

\[ \text{Re}_p = \frac{|\vec{V}_f - \vec{V}_p|d_p}{v} , \]  

(14)

and \( v \) is the fluid kinematics viscosity.

For the particle’s trajectory, its position is given by

\[ \frac{d \vec{X}_p}{dt} = \vec{V}_p . \]  

(15)

The particle’s velocity and position can also be obtained by integrating Eqs. (12) and (15) using the second-order Runge-Kutta scheme.

**Simulation of gas-particle two phase mixing layer flow**

For the plane mixing layer flow as shown schematically in Fig. 1, two parallel semi-infinite streams of velocities \( U_1 \) and \( U_2 \) are separated by a thin splitter plate and merge at

![Fig. 1. Planar mixing layer](image-url)