H2-Control and the Separation Principle for Discrete-Time Markovian Jump Linear Systems

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Abstract. In this paper we consider the H2-control problem of discrete-time Markovian jump linear systems. We assume that only an output and the jump parameters are available to the controller. It is desired to design a dynamic Markovian jump controller such that the closed-loop system is mean square stable and minimizes the H2-norm of the system. As in the case with no jumps, we show that an optimal controller can be obtained from two sets of coupled algebraic Riccati equations, one associated with the optimal control problem when the state variable is available, and the other associated with the optimal filtering problem. This is the principle of separation for discrete-time Markovian jump linear systems. When there is only one mode of operation our results coincide with the traditional separation principle for the H2-control of discrete-time linear systems.

Key words. Discrete-time, H2-control, Infinite-horizon, Markovian jump systems, Separation principle.

1. Introduction

Modern control systems must meet performance requirements and maintain acceptable behavior even in the presence of abrupt changes in their dynamics due, for instance, to random component failures or repairs, abrupt environmental disturbances, changes in subsystem interconnections, abrupt changes in the operation point of a non-linear plant, etc. Examples of such systems can be found, for instance, in the control of solar thermal central receivers, robotic manipulator systems, aircraft control systems, economic systems, large flexible structures for space stations (such as antenna, solar arrays), etc. A common way for analyzing systems in this class is through the multiple model approach, which assumes that the real

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system can be represented by a finite number of possible models. A usual hypothesis is to assume that the models are discrete-time linear systems and transitions among them follow a Markov chain \( \theta(k) \in \{1, \ldots, N\} \). The possible values of the Markov parameter characterizes the “operation modes” of the system. This class of systems is known as Markovian jump linear systems (MJLS), and has been extensively studied in several papers, for instance, [1], [2], [3], [4], [5], [6], [7], and [8]–[12].

We consider throughout this paper that the current operation mode \( \theta(k) \) is known at each time \( k \). Although in engineering problems the operation modes are not often available, there are enough cases where knowledge of random changes in system structure is directly available to make these applications of great interest. This is the case, for instance, of a non-linear plant approximated by a countable number of operating points, each of them characterized by a corresponding linearized model, and the abrupt changes would represent the dynamics of the system moving from one operation point to another. In many situations it is possible to monitor these changes in the operating conditions of the process through appropriate sensors. In a deterministic formulation, an adaptive controller that changes its parameters in response to the monitored operating conditions of the process is termed a gain scheduling controller (see Chapter 9 of [13]). That is, it is a linear feedback controller whose parameters are changed as a function of operating conditions in a preprogrammed way. Several examples of this kind of controller are presented in [13], and they could also be seen as examples for the optimal control problem of systems subject to abrupt dynamic changes, with the operation mode representing the monitored operation condition, and transition between the models following a Markov chain. Another possibility of systems subject to abrupt changes would be due to random failures/repairs of the process, with \( \theta(k) \) in this case indicating the nature of any failure. For \( \theta(k) \) to be available, an appropriate failure detector (see [14]) would be used in conjunction with a control reconfiguration given by the optimal control of an MJLS. In this context an interesting problem would be to work with an estimate \( \hat{\theta}(k) \) of the real value \( \theta(k) \). In [4] conditions on the probability of a correct reading of \( \theta(k) \) (that is, on \( \mathcal{P}(\theta(k) = i \mid \theta(k) = i) \) were established in order to have stability of the closed-loop system of an MJLS when we replace the real value \( \theta(k) \) by its estimate \( \hat{\theta}(k) \). We believe that one possible continuation of the presented work would be to consider, as in [4], controllers based on \( \hat{\theta}(k) \) instead of \( \theta(k) \). We present an informal discussion of these ideas in Section 7.

An example of a feedback control for an MJLS was presented in [15]. In this paper the authors considered the control of a solar-powered boiler, with the operation modes representing abrupt environmental changes measured by sensors located on the plant. The boiler flow rate is strongly dependent upon the receiving insolation and, as a result of this abrupt variability, several linearized models are required to characterize the evolution of the boiler when clouds interfere with the sun’s rays. The control law described in that paper makes use of the state feedback and a measurement of the operation mode through the use of flux sensors on the receiver panels. We return to this example in Section 6, when we consider a version of this problem with noisy state measures and infinity horizon quadratic