Abstract The readability of fuzzy models is related to their organizational structure and the corresponding rule base. On this basis, a new methodology for organizing the information, the Separation of Linguistic Information Methodology (SLIM), is developed. Based on its results, different algorithms are presented for different structures: the Parallel Collaborative Structure (PCS) - SLIM-PCS algorithm and the Hierarchical Prioritized Structure (HPS), SLIM-HPS algorithm. Finally, it is proposed a Fuzzy Clustering of Fuzzy Rules Algorithm (FCFRA) that allows the automatic organization of the sets of fuzzy IF ... THEN rules of one fuzzy system in a Parallel Collaborative Structure, the probabilistic Fuzzy Clustering, and in a Hierarchical Prioritized Structure, the Possibilistic Fuzzy Clustering.

Keywords Relevance · Hierarchical fuzzy model · Clustering of fuzzy sets

Introduction

Fuzzy system modelling provides a framework for the representation of information about complex relationships between variables. An important feature of this approach is the use of granularity and gradualarity [1], where the base elements are fuzzy if-then rules. These rules are essentially fuzzy relationships of the model, and with the aid of an inference mechanism it is possible to manipulate the information contained in the fuzzy rule base to infer new knowledge implicitly contained in the model. Compared to traditional mathematical modelling and pure neural network modelling, fuzzy modelling presents some distinctive advantages, such as the use of the mechanism of reasoning in human understandable terms and the capacity of taking linguistic information from human experts. Given a sufficient number of rules, the fuzzy logic system, FLS, can approximate an unknown mapping or function with any desired complexity over a compact domain [2–6]. As a universal approximator, the FLS have been successfully applied to solve many real-world problems [7–12].

However, there are several difficulties in using single, usually large, monolithic systems to solve complex problems. On problems involving a large number of features or input variables, the complexity of fuzzy system (in terms of number of rules) must be increased. In this situation, the centralized manner of computing by a monolithic fuzzy system results on a large and unintelligible computation overhead. Moreover, most existing fuzzy modelling approaches concentrate on model accuracy, simply fitting the data with the highest possible accuracy while paying little attention to simplicity and interpretability of the models obtained, which are considered the primary merits of fuzzy rule-based systems. Often users require the model not only to predict the system’s output accurately but also to provide a useful description of the system that generated the data, i.e., it is desirable that fuzzy models provide both satisfactory accuracy and good interpretation capability. A natural answer for such complex problems is to employ multiple smaller fuzzy system modules in a distributed fashion, instead of a single, large, monolithic, centralized and opaque fuzzy system.

Motivated by these considerations, the present work aims to organize the fuzzy system by making the distribution of functionalities between multiples fuzzy systems modules that process same or different information, in order to achieve improved computation and transparency. In order to organize fuzzy rules, it is proposed a metrics that quantify the relative importance of different sets of rules that describe a region of the input/output space in a fuzzy system, which are not directly related to its contribution to the minimization of the output error.
This new fuzzy rules metrics is the result of a new concept of relevance of fuzzy rules and of fuzzy system, recently proposed by Salgado [13]. This key concept provides a measure of the relative importance of sets of rules in the description of a given region of the input/output space. The spread of definition of relevance in the boundary regions and by the fuzzy system are proposed here.

These new concepts, bounded by a set of intuitive axioms, open the doors to new types of fuzzy systems. These axioms lead to a set of properties that are analyzed in some detail in the rest of this paper.

In order to organize the fuzzy systems into news structures, a new methodology called Separation of Linguistic Information Methodology [SLIM] [13, 14], is proposed. It is useful for organizing the information in a fuzzy system: a system \( f(x) \) is organized as a set of \( n \) fuzzy sub-systems \( f_1(x), f_2(x), \ldots, f_n(x) \). Each of these systems may contain information related to particular aspects of the system \( f(x) \).

In this paper, we propose two news structures resulting from the decomposition of the monolithic fuzzy system using the SLIM methodology: Hierarchical Prioritized Structure (HPS), which allows the information to be organized in the prioritized fashion [15–17] and the Parallel Collaborative Structure (PCS), where each model collaborates equally with the other models. With the HPS structure, Yager has introduced a new perspective: instead of a fuzzy system consisting of a set of rules with no ordering, apparently all with the same relative importance, priorities are defined, reflecting to the importance of the rules in the description of the process being modelled. The rule-ordering method suggested by Yager is based on the comparison of each pair of rules in the system. This leads to the establishment of binary relations between the rules. If the relations are well behaved, some results from preference theory [18], can be used to define a ranking order for the rules. However, the proposed method is not appropriate for situations where the number of rules is large. In contrast, in the PCS, each system both works independently and its decisions are combined by a fusion centre, which takes into account the relevance firing values.

The application of the SLIM methodology in these structures is used to organize the information, by exchanging information between various layers of the structure. It is also possible to reduce the number of rules representing the original system by discarding rules with lower relevance values.

Finally, two algorithms that implement Fuzzy Clustering of Fuzzy Rules (FCFR) are presented. By using the proposed algorithm, it is possible to group a set of rules in \( c \) subgroups (clusters) of similar rules. This provides a generalization of the fuzzy \( c \)-means clustering algorithm, applied to rules instead of points in \( R^d \). With this algorithm, the system obtained from the data is transformed into a new system, organized in subsystems, which are structured either along PCS or HPS lines.

The SLIM decomposition algorithms, along with the resulting HPS or PCS structures, are applied to solving two real-world problems. The first problem is a benchmark problem of efficient manner to describing a Volcano surface by a fuzzy model, early used to test other SLIM strategies algorithm [14], while the second is a complex real modelling problem of the Environmental Greenhouse model. This last work demonstrates the potential advantage of this new methodology to find and study real sub-models.

The paper is organized as follows. The concept of the relevance of a set of rules is defined in Sect. 2. The SLIM methodology and different structures (HPS and PCS) are discussed in Sect. 3. Brief reviews of early SLIM algorithms are made. In Sect. 4 news FCFR strategies are proposed. Various examples and experimental tests are presented in Sect. 5. Finally, the main conclusions are outlined in Sect. 6.

### The concept of relevance

#### Relevance in well-defined support region \( S \)

Fuzzy system modelling is a technique for modelling complex nonlinear relationships using a rule-based methodology. Central to this approach is a partitioning of the input/output space by fuzzy rules, which map regions in an input space, \( U \), to regions in an output space, \( V \), describing a region in a product space \( S = U \times V \). In fuzzy systems modelling we represent this relationship by a collection of fuzzy rules of the form

\[ R^l : \text{If } U \text{ is } \mu^l(x_k) \text{ then } V \text{ is } \theta^l \]

The \( \mu^l \)'s and \( \theta^l \)'s are normal fuzzy subsets over the spaces \( U \) and \( V \). In this relationship, the contributions of the different rules will be unequal. One main question will be formulated: “How to measure the relative importance of the rules that describe the region \( S' \)?”. Moreover, the fuzzy system cannot completely describe the region \( S \), raising the issue of how to measure the quality of fuzzy system in describing the \( S \) region? or whether the region \( S \) is perfectly described by the fuzzy system? The concept of relevance is designed to help clarify those questions.

The adoption of fuzzy logic theory and focusing on the degree of relevance as an extended fuzzy measure seems to be an appropriate approach to automating relevance perception of the rules and fuzzy systems. In [13], the following definition was proposed for the relevance of a set of rules.

### Definition 1.

Consider \( \exists \) a set of rules from \( U \) into \( V \), covering the region \( S = U \times V \) in the product space. The relevance of the rule \( R \in \mathcal{P}(\exists) \) on a region \( S \) can be characterized by attributing a real positive value to the rule. The normalized relevance function maps the power set of fuzzy rules \( \mathcal{P}(\exists) \) on the real interval \([0, 1]\), i.e., \( \mathbb{R}(R) \in [0, 1] \).