Wind-induced baroclinic response of Lake Constance

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Abstract. We present results of various circulation scenarios for the wind-induced three-dimensional currents in Lake Constance, obtained with the aid of a semi-spectral semi-implicit finite difference code developed in Haidvogel \textit{et al.} and Wang and Hutter. Internal Kelvin and Poincaré-type oscillations are demonstrated in the numerical results, whose periods depend upon the stratification and the geometry of the basin and agree well with measured data. By solving the eigenvalue problem of the linearized shallow water equations in the two-layered stratified Lake Constance, the interpretation of the oscillations as Kelvin and Poincaré-type waves is corroborated.

Key words: Oceanography: general (limnology; numerical modeling) – Oceanography: physical (internal and inertial waves)

1 Introduction

The stratification in lakes is almost exclusively established by the heat input due to solar radiation. During the summer season, in temperate climate zones, a stratification given at one particular instant generally persists for a time span that is long in comparison to the time scales of the dynamic circulation which is established by the wind shear traction applied at the water surface. Only during short-lived episodes, when the shearing in the surface layer is very large (i.e., typically during strong storms), Kelvin-Helmholtz instabilities can form which then generate internal bores that destroy the stratification or may erode the thermocline. During the periods in between the change in mass distribution due to circulation dynamics is relatively small. Thus, a stable underlying stratification with no movement may be assumed as an initial condition from which the temperature and velocity field may develop without forming internal instabilities.

The underlying thermo-mechanical processes can be described mathematically by the shallow water equations in the Boussinesq approximation: in these equations the vertical momentum balance reduces to the force balance between pressure gradient and gravity (buoyancy) force, and density variations are only accounted for in the buoyancy term. The equations describe both, barotropic and baroclinic processes. The former are those that develop when density variations do not exist and, in temperate zones, arise in late autumn and during winter. They also exist in a truly stratified lake or ocean basin, but are then overshadowed by the baroclinic processes that are typical for the summer seasons when the underlying stratification is strong.

There are many numerical models based on the shallow water equations in the Boussinesq approximation (Ramming and Kowalik, 1980; Simons, 1980; Oman, 1982; Pohlmann, 1987; Tee, 1987; Lehmann, 1995). Haidvogel and Beckmann (1998) also summarized many such three-dimensional models. All claim to describe these kind of wind-induced motions, and have been applied to large-scale oceanographic situations. Some three-dimensional models have been used to describe circulation flows in lakes and have had limited success (e.g. Bennett, 1977; Hollan and Simons, 1978; Oman, 1982). However these three-dimensional models or codes are limited in their applicability, or unsatisfactory, because internal wave processes are overly damped owing to the large explicit or implicit numerical diffusion that had to be built into the codes to stabilize them under common conditions.

Here, the nonlinear shallow water equations in the Boussinesq approximation, that describe wind induced circulation dynamics in enclosed basins, are integrated using a numerical semi-spectral primitive equation model (SPEM) that was modified by us with a semi-implicit time-integration in the vertical direction. For greater detail the reader is referred to Wang and Hutter.

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(1998). Here our aim is the presentation of solutions to a
given initial stratification and a wind stress from
different directions.

In Sect. 2 we briefly outline the model. In Sect. 3
various scenarios of wind-driven baroclinic processes in
Lake Constance are studied and the predicted Kelvin
and Poincaré-type oscillations can be displayed in the
results. In Sect. 4, by solving the eigenvalue problem of
the linearized shallow water equations of the two-
layered Lake Constance, the interpretation of the
oscillations as Kelvin and Poincaré-type waves can be
ascertained. We conclude with a summary and some
remarks in Sect. 5.

2 Numerical model

We briefly describe here the numerical model as
constructed by Haidvogel et al. (1991) and changed by
Wang and Hutter (1998) to allow large-time-temporal
integration.

The basic hydrodynamic equations consist of the
balance equations of mass, momentum and energy as
well as a thermal equation of state. We apply these
hydrodynamic equations in the shallow water and
Boussinesq approximations, with the Coriolis term and
the hydrostatic pressure equation implemented. A fur-
ther simplification is achieved by imposing a rigid lid at
the water surface. Under these assumptions the field
equations read

\[ \frac{\partial u}{\partial t} + \nabla \cdot v = 0, \]
\[ \frac{\partial u}{\partial t} + v \cdot \nabla u + \frac{\partial}{\partial z} = - \frac{\partial \phi}{\partial x} + \frac{\partial}{\partial x} \left( \frac{v_H}{\partial x} \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{v_H}{\partial y} \frac{\partial u}{\partial y} \right) + \frac{\partial}{\partial z} \left( \frac{v_H}{\partial z} \frac{\partial u}{\partial z} \right), \]
\[ \frac{\partial v}{\partial t} + v \cdot \nabla v + \frac{\partial}{\partial z} = - \frac{\partial \phi}{\partial y} + \frac{\partial}{\partial x} \left( \frac{v_H}{\partial x} \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{v_H}{\partial y} \frac{\partial v}{\partial y} \right) + \frac{\partial}{\partial z} \left( \frac{v_H}{\partial z} \frac{\partial v}{\partial z} \right), \]
\[ 0 = - \frac{\partial \phi}{\partial z} - \frac{\rho g}{\rho_0}, \]
\[ \frac{\partial T}{\partial t} + v \cdot \nabla T = - \frac{\partial}{\partial x} \left( \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( \frac{\partial T}{\partial z} \right), \]
\[ \frac{\rho - \rho_0}{\rho_0} = - \alpha (T - T_0)^2, \quad \text{in °C} \]

Here a Cartesian coordinate system \((x, y, z)\) has been used;
\((x, y)\) are horizontal, and \(z\) is vertically upwards, against
the direction of gravity; \(v = (u, v, w)\), \(f\), \(\rho\), \(\rho_0\), \(\phi\), \(g\), \(T\), \(z\)
are, respectively, the velocity vector, Coriolis parameter,
density, reference density \((\rho_0 = 1000 \text{ kg m}^{-3})\) at
temperature \(T_0 = 4^\circ\text{C}\), dynamic reduced pressure \((\phi = p/\rho_0, p\) is
pressure), gravity force \((g = 9.8 \text{ m s}^{-2})\), temperature,
thermal expansion coefficient. For Lake Constance (fresh
water) \(z\) is taken as \(z = 6.8 \times 10^{-6} (\text{C})^{-2}\). Furthermore,
\(v_H, v_V\) are horizontal and vertical momentum, \(D_H, D_V\),
horizontal and vertical heat diffusivities.

To solve the system of differential equations numeri-
cally, a semi-spectral model was designed with semi-
implicit integration in time. The model is based on the
semi-spectral model SPEM developed by Haidvogel
et al. (1991), in which the vertical dependence of the
model variables is represented as an expansion in a finite
modified Chebyshev polynomial set, and in the hori-
zontal finite difference representations are used. SPEM
employs in its original version an explicit scheme for
temporal integration and thus is numerically only
conditionally stable, i.e., the allowable time step is
restricted by spatial resolutions. Therefore this model
was extended by Wang and Hutter (1998) to account for
implicit temporal integration: because of the small water
depths of lakes in comparison to the ocean, the original
SPEM model had to be altered to permit economically
justifiable time steps in the computation of the circula-
tion of a lake. In Wang and Hutter (1998) several finite
difference schemes, implicit in time, were introduced;
that scheme which uses implicit integration in time for
the viscous terms in the vertical direction was the most
successful one.

By using a \(\sigma\)-transformation, a lake domain with
varying topography is transformed to a new domain
with constant depth, and this region is once again
transformed in the horizontal co-ordinates by using
conformal mapping which maps the shore as far as possible
onto a rectangle. Uniformity in grid size distribution
is intended, because numerical oscillations (instabilities)
preferably occur on the small scale; however,
it is difficult to achieve it in complex geometries. In
such cases, to attain as far as possible an uniform grid
distribution, a bounding line, which deviates in some
segments from the actual lake boundary, is used for the
conformal mapping. In these segments the actual
boundaries can only be approximated by a step func-
tion, and in the numerical computations the land areas
within the grid system must be excluded by a special
masking technique (Wilkin et al., 1995).

This numerical code has proved its suitability in
several lake applications, including diffusion problems
(Hutter and Wang, 1998), substructuring procedures
(Chubarenko et al., 1999; Wang and Hutter, 2000) and
wave dynamics in wind-driven circulation (Hutter et al.,
1998). For this reason we refrain from prescribing the
numerical code and directly pass on to the applica-
tion of this code to baroclinic motions in Lake Constance.

3 Baroclinic response in Lake Constance

3.1 Parameter selection

Lake Constance (Bodensee, Fig. 1a), the second largest
Alpine lake in Europe, consists of three basins, the main
basin of Upper Lake Constance and Lake Überlingen
which together form Upper Lake Constance (Obersee),
and the Lower Lake Constance (Untersee); the latter is
dynamically decoupled from the others by the 5 km long