Low frequency waves in plasmas with spatially varying electron temperature

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Abstract. Low frequency electrostatic waves are studied in magnetized plasmas with an electron temperature which varies with position in a direction perpendicular to the magnetic field. For wave frequencies below the ion cyclotron frequency, the waves need not follow any definite dispersion relation. Instead a band of phase velocities is allowed, with a range of variation depending on the maximum and minimum values of the electron temperature. Simple model equations are obtained for the general case which can be solved to give the spatial variation of a harmonically time varying potential. A simple analytical model for the phenomenon is presented and the results are supported by numerical simulations carried out in a 2-dimensional particle-in-cell numerical simulation. We find that when the electron temperature is striated along B₀ and low frequency waves (ω ≪ Ω_e) are excited in this environment, then the intensity of these low frequency waves will be striated in a manner following the electron temperature striations. High frequency ion acoustic waves (ω ≫ Ω_e) will on the other hand have a spatially more uniform intensity distribution.

Key words: Ionosphere (plasma temperature and density) – Radio science (waves in plasma) – Space plasma physics (numerical simulation studies)

1 Introduction

The propagation of low frequency electrostatic waves in magnetized plasmas represents a classic study in low temperature laboratory plasmas. In one of the most versatile experimental set-ups, the Q-machine, extensive investigations were reported on the propagation of low frequency ion sound waves along an externally imposed magnetic field, i.e. in an essentially one dimensional geometry (Motley, 1975). Ideally, the conditions in the direction transverse to the magnetic field were assumed constant, but in reality the plasma density as well as the ion and electron temperatures can vary in these experiments and in similar ones. Also, it may be a rule rather than an exception that the electron as well as the ion temperatures are spatially inhomogeneous in naturally occurring plasmas out of equilibrium (Peñano et al., 2000). In the present study we investigate some consequences of a spatially varying electron temperature. The study is carried out in a slab geometry with a homogeneous magnetic field along the z-axis and T_e = T_e(x). This electron temperature variation is assumed to be given a priori. We find, for instance, that in cases where the electron temperature is striated along B₀ and low frequency waves (ω ≪ Ω_e) are excited in this environment, for instance by a plasma instability, then the intensity of these low frequency waves will be striated in a manner following the electron temperature striations. High frequency ion acoustic waves (ω ≫ Ω_e) will, on the other hand, have a spatially more uniform intensity distribution. These effects should be readily noticeable by, for instance, an instrumented space craft. The basic physical principles underlying the analysis are quite simple, and the phenomena described in the present work can have importance for the interpretation of, e.g. ionospheric or magnetospheric plasma phenomena.

2 Analytical results

For the present study we are primarily interested in waves with frequency well below the ion cyclotron frequency. However, this problem contrasts in an interesting way with its high frequency counterpart. Both cases are therefore discussed. In the linearized analysis used in the present study, a density gradient will be immaterial, since the linear sound velocity does not
depend on the plasma density. The investigations are therefore mainly concerned with variations in the electron temperature.

2.1 The low frequency case, $\omega < \Omega_e$

In this section we consider a simple analytical model which seems to account for the essential parts of our findings. The model assumes a vanishing ion temperature $T_i = 0$ in the quasi-neutral limit where $n_e \approx n_i \equiv n$. The basic equations are the ion continuity and momentum equations as well as the $z$-component of the electron momentum equation where the electron inertia has been neglected. These equations are respectively, in their linearized form

\[ \frac{\partial n}{\partial t} + n_0 \mathbf{V} \cdot \mathbf{u} = 0 \]  
\[ M \frac{\partial \mathbf{u}}{\partial t} - e( - \mathbf{V} \phi + \mathbf{u} \times \mathbf{B}_0) = 0 \]  
\[ -en_0 \frac{\partial \phi}{\partial z} + \frac{\partial}{\partial z} [nT_e(x)] = 0 \ , \]

where $n$ and $\mathbf{u}$ are the density and velocity perturbations, $T_e$ is the electron temperature, $\phi$ the electrostatic potential ($\mathbf{E} = -\mathbf{V} \phi$), $\mathbf{B}_0$ the magnetic field, $e$ and $M$ the ion charge and mass, respectively. Assuming that the relevant frequencies are well below the ion-cyclotron frequency $\Omega_e$, we iterate Eq. (2) in order to obtain the standard approximation for the ion velocity component perpendicular to the magnetic field

\[ \mathbf{u}_\perp = -\frac{\mathbf{V}_\perp \phi \times \mathbf{B}_0}{B_0^2} - \frac{1}{B_0 \Omega_e} \frac{\partial}{\partial t} \frac{\partial \phi}{\partial z_\perp} \ , \]

The first term is the $\mathbf{E} \times \mathbf{B}$ drift velocity and the second term the ion polarization drift. With a little standard algebra and in the relevant two spatial dimensions $(x, z)$, we obtain a differential equation for the electrostatic potential

\[ \frac{e}{T_e(x)} \frac{\partial^2 \phi}{\partial t^2} - \frac{1}{B_0 \Omega_e} \frac{\partial}{\partial x} \frac{\partial^2 \phi}{\partial z^2} - \frac{e}{M} \frac{\partial^2 \phi}{\partial z^2} = 0 \ . \]

The $\mathbf{B}_0$-perpendicular component of the electron dynamics does not enter the analysis with the assumption of Boltzmann distributed electrons, see also a brief discussion in Appendix A. As mentioned, we here assume that the ion temperature is vanishing for simplicity. The results for $T_i \neq 0$ are not significantly different, as demonstrated in Appendix B.

For a homogeneous plasma with constant electron temperature $T_e$, we find the standard dispersion relation

\[ \omega^2 = \frac{k_z^2 C_s^2}{1 + k_z^2 C_s^2 / \Omega_e^2} \]

for $\omega^2 \ll \Omega_e^2$, where $C_s = \sqrt{T_e / M}$ is the ion sound acoustic velocity and $C_s / \Omega_e$ takes the role of an effective Larmor radius. The dispersion relation is shown in Fig. 1 (upper panel), together with a vectorial presentation of the group velocity $v_g$, deduced from Eq. (6) (lower panel). We note that $k \cdot v_g = k_z C_s / (1 + k_z^2 C_s^2 / \Omega_e^2)^{3/2}$, and the directions of $k$ and $v_g$ are different. In Fig. 2 we show the variation of the angle between the wave vector and the group velocity for varying wave vector components, $k_x$ and $k_z$. Retaining the spatially varying electron temperature, we can still take the Fourier transform in time and in space, along the $z$-axis, to obtain

\[ \text{Fig. 2. The variation of the angle between the wave vector and the group velocity for varying wave vector components, } k_x \text{ and } k_z \text{. The point } (k_x, k_z) = (0, 0) \text{ is singular. The results apply for } \omega^2 \ll \Omega_e^2 \]